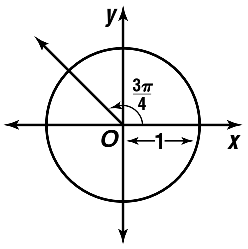
Chapter 6 Graphs of Trigonometric Functions

6-1 Angles and Radian Measure

Pages 347–348 Check for Understanding

1.



2. $90°; \frac{\pi}{4}$

3. Divide 10 by 8.

4. Let $R = 2r$. For the circle with radius R, $s' = R\theta$ or $2r\theta$ which is $2(r\theta)$. Thus, $s' = 2s$. For the circle with radius R, $A' = \frac{1}{2}R^2\theta$ or $\frac{1}{2}(2r)^2\theta$ which is $\frac{1}{2}(4r^2)\theta$ or $4\left(\frac{1}{2}r^2\theta\right)$. Thus, $A' = 4A$.

5. $240° = 240° \times \frac{\pi}{180°} = \frac{4\pi}{3}$

6. $570° = 570° \times \frac{\pi}{180°} = \frac{19\pi}{6}$

7. $\frac{3\pi}{2} = \frac{3\pi}{2} \times \frac{180°}{\pi} = 270°$

8. $-1.75 = -1.75 \times \frac{180°}{\pi} = 100.3°$

9. reference angle: $\frac{3\pi}{4} - \pi$ or $\frac{\pi}{4}$; Quadrant 2
$\sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$

10. reference angle: $\frac{11\pi}{6} - \pi$ or $\frac{5\pi}{6}$; Quadrant 3
$\tan \frac{11\pi}{6} = -\frac{\sqrt{3}}{3}$

11. $s = r\theta$
$s = 15\left(\frac{5\pi}{6}\right)$
$s \approx 39.3$ in.

12. $77° = 77° \times \frac{\pi}{180°} = \frac{77\pi}{180}$
$s = r\theta$
$s = 15\left(\frac{77\pi}{180}\right)$
$s \approx 20.2$ in.

13. $A = \frac{1}{2}r^2\theta$
$A = \frac{1}{2}(1.4^2)\left(\frac{2\pi}{3}\right)$
$A \approx 2.1$ units^2

14. $54° = 54° \times \frac{\pi}{180°} = \frac{3\pi}{10}$
$A = \frac{1}{2}r^2\theta$
$A = \frac{1}{2}(6^2)\left(\frac{3\pi}{10}\right)$
$A \approx 17.0$ units^2

15. $30° = 30° \times \frac{\pi}{180°} = \frac{\pi}{6}$
$s = r\theta$
$s = 1.4\left(\frac{\pi}{6}\right)$
$s \approx 0.7$ m

Pages 348–351 Exercises

16. $135° = 135° \times \frac{\pi}{180°} = \frac{3\pi}{4}$

17. $210° = 210° \times \frac{\pi}{180°} = \frac{7\pi}{6}$

18. $300° = 300° \times \frac{\pi}{180°} = \frac{5\pi}{3}$

19. $-450° = -450° \times \frac{\pi}{180°} = -\frac{5\pi}{2}$

20. $-75° = -75° \times \frac{\pi}{180°} = -\frac{5\pi}{12}$

21. $1250° = 1250° \times \frac{\pi}{180°} = \frac{125\pi}{18}$

22. $\frac{7\pi}{12} = \frac{7\pi}{12} \times \frac{180°}{\pi} = 105°$

23. $\frac{11\pi}{3} = \frac{11\pi}{3} \times \frac{180°}{\pi} = 660°$

24. $17 = 17 \times \frac{180°}{\pi} = 974.0°$

25. $-3.5 = -3.5 \times \frac{180°}{\pi} = -200.5°$

26. $-\frac{\pi}{6.2} = -\frac{\pi}{6.2} \times \frac{180°}{\pi} = -29.0°$

27. $17.5 = 17.5 \times \frac{180°}{\pi} = 1002.7°$

28. reference angle: $2\pi - \frac{5\pi}{3}$ or $\frac{\pi}{3}$; Quadrant 4
$\sin \frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$

29. reference angle: $\frac{7\pi}{6} - \pi$ or $\frac{\pi}{6}$; Quadrant 3
$\tan \frac{7\pi}{6} = \frac{\sqrt{3}}{3}$

30. reference angle: $\frac{5\pi}{4} - \pi$ or $\frac{\pi}{4}$; Quadrant 3
$\cos \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$

31. reference angle: $\frac{7\pi}{6} - \pi$ or $\frac{\pi}{6}$; Quadrant 3
$\sin \frac{7\pi}{6} = -\frac{1}{2}$

32. $\frac{14\pi}{3}$ is coterminal with $\frac{2\pi}{3}$
reference angle: $\pi - \frac{2\pi}{3}$ or $\frac{\pi}{3}$; Quadrant 2
$\tan \frac{14\pi}{3} = -\sqrt{3}$

33. $-\frac{19\pi}{6}$ is coterminal with $\frac{5\pi}{6}$
reference angle: $\pi - \frac{5\pi}{6}$ or $\frac{\pi}{6}$; Quadrant 2
$\cos\left(-\frac{19\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

34. $s = r\theta$
$s = 14\left(\frac{2\pi}{3}\right)$
$s \approx 29.3$ cm

35. $s = r\theta$
$s = 14\left(\frac{5\pi}{12}\right)$
$s \approx 18.3$ cm

36. $150° = 150° \times \frac{\pi}{180°} = \frac{5\pi}{6}$
$s = r\theta$
$s = 14\left(\frac{5\pi}{6}\right)$
$s \approx 36.7$ cm

37. $282° = 282° \times \frac{\pi}{180°} = \frac{47\pi}{30}$
$s = r\theta$
$s = 14\left(\frac{47\pi}{30}\right)$
$s \approx 68.9$ cm

38. $s = r\theta$
$s = 14\left(\frac{3\pi}{11}\right)$
$s \approx 12.0$ cm

39. $320° = 320° \times \frac{\pi}{180°} = \frac{16\pi}{9}$
$s = r\theta$
$s = 14\left(\frac{16\pi}{9}\right)$
$s \approx 78.2$ cm

40. $r = \frac{1}{2}d$

$r = \frac{1}{2}(22)$

$r = 11$

$78° = 78° \times \frac{\pi}{180°}$

$= \frac{13\pi}{30}$

$s = r\theta$

$s = 11\left(\frac{13\pi}{30}\right)$

$s \approx 15.0$ in.

41. $s = r\theta$

$70.7 = r\left(\frac{5\pi}{4}\right)$

$18.00360716 \approx r$

$d = 2r$

$d \approx 2(18.0)$

$d \approx 36.0$ m

42. $60° = 60° \times \frac{\pi}{180°}$

$= \frac{\pi}{3}$

$s = r\theta$

$14.2 = r\left(\frac{\pi}{3}\right)$

$13.56 \approx r$; about 13.6 cm

43. $A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2}(10^2)\left(\frac{5\pi}{12}\right)$

$A \approx 65.4$ units^2

44. $90° = 90° \times \frac{\pi}{180°}$

$= \frac{\pi}{2}$

$A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2}(22^2)\left(\frac{\pi}{2}\right)$

$A \approx 380.1$ units^2

45. $A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2}(7^2)\left(\frac{\pi}{8}\right)$

$A \approx 9.6$ units^2

46. $A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2}(12.5^2)\left(\frac{4\pi}{7}\right)$

$A \approx 140.2$ units^2

47. $225° = 225° \times \frac{\pi}{180°}$

$= \frac{5\pi}{4}$

$A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2}(6^2)\left(\frac{5\pi}{4}\right)$

$A \approx 70.7$ units^2

48. $82° = 82° \times \frac{\pi}{180°}$

$= \frac{41\pi}{90}$

$A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2}(7.3^2)\left(\frac{41\pi}{90}\right)$

$A \approx 38.1$ units^2

49a. $s = r\theta$

$6 = r(1.2)$

$5 = r$; 5 ft

49b. $A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2}(5^2)(1.2)$

$A = 15$ ft^2

50a. $135° = 135° \times \frac{\pi}{180°}$

$= \frac{3\pi}{4}$

$s = r\theta$

$114 = r\left(\frac{3\pi}{4}\right)$

$48.38 \approx r$; about 48.4 mm

50b. $A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2}(48.4^2)\left(\frac{3\pi}{4}\right)$

$A \approx 2757.8$ mm^2

51a. $A = \frac{1}{2}r^2\theta$

$15 = \frac{1}{2}r^2(0.2)$

$150 = r^2$

$12.247 \approx r$

about 12.2 in.

51b. $s = r\theta$

$s \approx 12.2(0.2)$

$s \approx 2.4$ in.

52a. $A = \frac{1}{2}r^2\theta$

$15.3 = \frac{1}{2}(3^2)\theta$

$3.4 = \theta$; 3.4 radians

52b. $3.4 = 3.4 \times \frac{180°}{\pi}$

$\approx 194.8°$

52c. $s = r\theta$

$s = 3(3.4)$

$s = 10.2$ m

53a. $225° = 225° \times \frac{\pi}{180°}$

$= \frac{5\pi}{4}$

$s = r\theta$

$s = 2\left(\frac{5\pi}{4}\right)$

$s \approx 7.9$ ft

53b. $s = r\theta$

$5 = 2\theta$

$2.5 = \theta$

$2.5 = 2.5 \times \frac{180°}{\pi}$

$\approx 143.2°$

54. $330° = 330° \times \frac{\pi}{180°}$

$= \frac{11\pi}{6}$

$s = r\theta$

$s = 2\left(\frac{11\pi}{6}\right)$

$s \approx 11.5$ in.

$s = r\theta$

$11.5 \approx 8\theta$

$1.44 \approx \theta$; about 1.4 radians

55. $s = r\theta$

$10.5 = 22.9\theta$

$0.46 \approx \theta$; about 0.5

$0.5 \approx 0.5 \times \frac{180°}{\pi}$

$\approx 26.3°$

56a. $45° - 34° = 11°$

$11° = 11° \times \frac{\pi}{180°}$

$= \frac{11\pi}{180}$

$s = r\theta$

$s = 3960\left(\frac{11\pi}{180}\right)$

$s \approx 760.3$ mi

56b. $45° - 31° = 14°$

$14° = 14° \times \frac{\pi}{180°}$

$= \frac{7\pi}{90}$

$s = r\theta$

$s = 3960\left(\frac{7\pi}{90}\right)$

$s \approx 967.6$ mi

56c. $34° - 31° = 3°$

$3° = 3° \times \frac{\pi}{180°}$

$= \frac{\pi}{60}$

$s = r\theta$

$s = 3960\left(\frac{\pi}{60}\right)$

$s \approx 207.3$ mi

57. $84.5° = 84.5° \times \frac{\pi}{180°}$

$= \frac{169\pi}{360}$

$s = r\theta$

$s = 0.70\left(\frac{169\pi}{360}\right)$

$s \approx 1.03$ mi

$80° = 80° \times \frac{\pi}{180°}$

$= \frac{4\pi}{9}$

$s = r\theta$

$s = 0.67\left(\frac{4\pi}{9}\right)$

$s \approx 0.94$ mi

$1.03 + 1.46 + 0.94 + 1.8 \approx 5.23$ mi

58a. $r = \frac{1}{2}d$

$r = \frac{1}{2}\left(2\frac{1}{2}\right)$

$r = 1.25$

$s = r\theta$

$s = 1.25(3\pi)$

$s \approx 11.8$ ft

1.5 rotations $= 1.5 \times 2\pi$ radians

$= 3\pi$ radians

58b. $s = r\theta$

$4\frac{1}{2} = 1.25\theta$

$3.6 = \theta$

$3.6 = 3.6 \times \frac{180°}{\pi}$

$\approx 206.3°$

59a. $\theta = 2\pi - \frac{\pi}{2}$ or $\frac{3\pi}{2}$

$A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2}(15^2)\left(\frac{3\pi}{2}\right)$

$A \approx 530.1$ ft^2

59b. $A = \frac{1}{2}r^2\theta$

$750 = \frac{1}{2}r^2\left(\frac{3\pi}{2}\right)$

$318.3098862 \approx r^2$

$17.84124116 \approx r$; about 17.8 ft

60. 3.5 km = 350,000 cm

$s = r\theta$

$350{,}000 = 32\theta$

$10{,}937.5 = \theta$; 10,937.5 radians

61. Area of segment = Area of sector − Area of triangle

$$A = \frac{1}{2}r^2\alpha - \frac{1}{2}r \cdot r \cdot \sin\alpha$$

$$A = \frac{1}{2}r^2(\alpha - \sin\alpha)$$

62. $s = \frac{1}{2}(6 + 8 + 12)$

$= 13$

$K = \sqrt{s(s-a)(s-b)(s-c)}$

$K = \sqrt{13(13-6)(13-8)(13-12)}$

$K = \sqrt{455}$

$K \approx 21.3$ in^2

63. Since $152° \geq 90°$, consider Case II.

$10.2 \leq 12$, so there is no solution.

64. $C = 180° - 38° - 27° = 115°$

$\frac{560}{\sin 115°} = \frac{a}{\sin 27°}$

$a \approx 280.52$

$\sin 38° \approx \frac{x}{280.52}$

$x \approx 172.7$ yd

65. I, III

66a. Find a quadratic regression line using a graphing calculator. Sample answer: $y = 102x^2 - 505x + 18{,}430$

66b. $2020 - 1970 = 50$

$y = 102x^2 - 505x + 18{,}430$

$y = 102(50)^2 - 505(50) + 18{,}430$

$y = 248{,}180$

Sample answer: about 248,180

67.

r	1	−3	−2	6	10
1	1	−2	−4	2	12
2	1	−1	−4	−2	6
3	1	0	−2	0	10
4	1	1	2	14	66

$f(-x) = (-x)^4 - 3(-x^3 - 2(-x)^2 + 6(-x) + 10$

$f(-x) = x^4 + 3x^3 - 2x^2 - 6x + 10$

r	1	3	−2	−6	10
1	1	4	2	−4	6
2	1	5	8	10	30

Sample answers: 4; −2

68.

−2	1	6	12	12
		−2	−8	−8
	1	4	4	4

No; there is a remainder of 4.

69. $x^2 + y^2 = 16 \rightarrow a^2 + b^2 = 16$

x-axis $\quad a^2 + b^2 = 16$

$a^2 + (-b)^2 = 16$

$a^2 + b^2 = 16$; yes

y-axis $\quad a^2 + b^2 = 16$

$(-a)^2 + b^2 = 16$

$a^2 + b^2 = 16$; yes

$y = x \quad a^2 + b^2 = 16$

$(b)^2 + (a)^2 = 16$

$a^2 + b^2 = 16$; yes

$y = -x \quad a^2 + b^2 = 16$

$(-b)^2 + (-a)^2 = 16$

$a^2 + b^2 = 16$; yes

all

70. $4x - 2y + 3z = -6$

$5x - 4y - 3z = -75$

$9x - 6y = -81$

$2(4x - 2y + 3z) = 2(-6)$
$3(3x + 3y - 2z) = 3(2)$ $\rightarrow$

$8x - 4y + 6z = -12$

$9x + 9y - 6z = 6$

$17x + 5y = -6$

$5(9x - 6y) = 5(-81)$
$6(17x + 5y) = 6(-6)$ $\rightarrow$

$45x - 30y = -405$

$102x + 30y = -36$

$147x = -441$

$x = -3$

$9x - 6y = -81$

$9(-3) - 6y = -81$

$y = 9$

$4x - 2y + 3z = -6$

$4(-3) - 2(9) + 3z = -6$

$z = 8$

$(-3, 9, 8)$

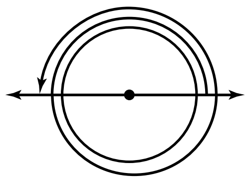
71. b

72. Since $q < 0$, $-q > 0$. Given that $p > 0$, $p - q = p + |q|$ and $p + |q| > 0$. So the expression $p - q$ is nonnegative.
The correct choice is B.

6-2 Linear and Angular Velocity

Page 355 Check for Understanding

1.



2. $\frac{5 \text{ rev}}{1 \text{ min}} \times \frac{2\pi \text{ radians}}{1 \text{ rev}} \times \frac{1 \text{ min}}{60 \text{ s}}$

3. Linear velocity is the movement along the arc with respect to time while angular velocity is the change in the angle with respect to time.

4. Both individuals would have the same change in angle during the same amount of time. However, an individual on the outside of the carousel would travel farther than an individual on the inside during the same amount of time.

5. Since angular velocity is $\frac{\theta}{t}$, the radius has no effect on the angular velocity. Let $R = 2r$. For a circle with radius R, $v' = R\frac{\theta}{t}$ or $(2r)\frac{\theta}{t}$ which is $2\left(r\frac{\theta}{t}\right)$. Thus $v' = 2v$.

6. $5.8 \times 2\pi = 11.6\pi$ or about 36.4 radians

7. $710 \times 2\pi = 1420\ \pi$ or about 4461.1 radians

8. $3.2 \times 2\pi = 6.4\pi$

$\omega = \frac{\theta}{t}$

$\omega = \frac{6.4\pi}{7}$

$\omega \approx 2.9$ radians/s

9. $700 \times 2\pi = 1400\pi$

$\omega = \frac{\theta}{t}$

$\omega = \frac{1400\pi}{15}$

$\omega \approx 293.2$ radians/min

10. $v = r\omega$
$v = 12(36)$
$v = 432$ in./s

11. $v = r\omega$
$v = 7(5\pi)$
$v \approx 110.0$ m/min

12a. $r = 3960 + 22{,}300$ or $26{,}260$ mi
$s = r\theta$
$s = 26{,}260(2\pi)$
$s \approx 164{,}996.4$ mi

12b. $v = r\frac{\theta}{t}$
$v = 26{,}260\left(\frac{2\pi}{24}\right)$
$v \approx 6874.9$ mph

Pages 355–358 Exercises

13. $3 \times 2\pi = 6\pi$ or about 18.8 radians

14. $2.7 \times 2\pi = 5.4\pi$ or about 17.0 radians

15. $13.2 \times 2\pi = 26.4\pi$ or about 82.9 radians

16. $15.4 \times 2\pi = 30.8\pi$ or about 96.8 radians

17. $60.7 \times 2\pi = 121.4\pi$ or about 381.4 radians

18. $3900 \times 2\pi = 7800\pi$ or about 24,504.4 radians

19. $1.8 \times 2\pi = 3.6\pi$
$\omega = \frac{\theta}{t}$
$\omega = \frac{3.6\pi}{9}$
$\omega \approx 1.3$ radians/s

20. $3.5 \times 2\pi = 7\pi$
$\omega = \frac{\theta}{t}$
$\omega = \frac{7\pi}{3}$
$\omega \approx 7.3$ radians/min

21. $17.2 \times 2\pi = 34.4\pi$
$\omega = \frac{\theta}{t}$
$\omega = \frac{34.4\pi}{12}$
$\omega \approx 9.0$ radians/s

22. $28.4 \times 2\pi = 56.8\pi$
$\omega = \frac{\theta}{t}$
$\omega = \frac{56.8\pi}{19}$
$\omega \approx 9.4$ radians/s

23. $100 \times 2\pi = 200\pi$
$\omega = \frac{\theta}{t}$
$\omega = \frac{200\pi}{16}$
$\omega \approx 39.3$ radians/min

24. $122.6 \times 2\pi = 245.2\pi$
$\omega = \frac{\theta}{t}$
$\omega = \frac{245.2\pi}{27}$
$\omega \approx 28.5$ radians/min

25. $\frac{1 \text{ revolution}}{50 \text{ seconds}} \times \frac{2\pi \text{ radians}}{1 \text{ revolution}} \approx 0.1$ radian/s

26. $\frac{500 \text{ revolutions}}{1 \text{ minute}} \times \frac{1 \text{ minute}}{60 \text{ seconds}} \times \frac{2\pi \text{ radians}}{1 \text{ revolution}} \approx 52.4$ radians/s

27. $\frac{85 \text{ radians}}{1 \text{ second}} \times \frac{60 \text{ seconds}}{1 \text{ minute}} \times \frac{1 \text{ revolution}}{2\pi \text{ radians}} \approx 811.7$ rpm

28. $v = r\omega$
$v = 8(16.6)$
$v = 132.8$ cm/s

29. $v = r\omega$
$v = 4(27.4)$
$v = 109.6$ ft/s

30. $v = r\omega$
$v = 1.8(6.1\pi)$
$v \approx 34.5$ m/min

31. $v = r\omega$
$v = 17(75.3\pi)$
$v \approx 4021.6$ in./s

32. $v = r\omega$
$v = 39(805.6)$
$v = 31{,}418.4$ in./min

33. $v = r\omega$
$v = 88.9(64.5\pi)$
$v \approx 18{,}014.0$ mm/min

34a. $\frac{120°}{1 \text{ second}} \times \frac{60 \text{ seconds}}{1 \text{ minute}} \times \frac{1 \text{ revolution}}{360°} = 20$ rpm

34b. $120° = 120° \times \frac{\pi}{180°}$
$= \frac{2\pi}{3}$

$\omega = \frac{\theta}{t}$
$\omega = \frac{\frac{2\pi}{3}}{1}$
$\omega = \frac{2\pi}{3}$

$v = r\omega$
$v = 5\left(\frac{2\pi}{3}\right)$
$v \approx 10.5$ in./s

35a. In 1 second, the second hand moves $\frac{1}{60}(360°)$ or 6°.
$6° = 6° \times \frac{\pi}{180°}$ or $\frac{\pi}{30}$
$v = r\theta$
$v = 30\left(\frac{\pi}{30}\right)$
$v \approx 3.1$ mm/s

35b. In 1 second, the minute hand moves $\frac{1}{60}\left(\frac{1}{60}\right)$ (360°) or 0.1°.
$0.1° = 0.1° \times \frac{\pi}{180°}$ or $\frac{0.1\pi}{180}$
$v = r\theta$
$v = 27\left(\frac{0.1\pi}{180}\right)$
$v \approx 0.05$ mm/s

35c. In 1 second, the hour hand moves $\frac{1}{12}\left(\frac{1}{60}\right)\left(\frac{1}{60}\right)$ (360°) or about 0.008°.
$0.008° = 0.008° \times \frac{\pi}{180°}$ or $\frac{0.008\pi}{180}$
$v = r\theta$
$v = 18\left(\frac{0.008\pi}{180}\right)$
$v \approx 0.003$ mm/s

36a. $r = \frac{1}{2}d$
$r = \frac{1}{2}(80)$ or 40

$v = r\frac{\theta}{t}$
$v = 40\left(\frac{2\pi}{45}\right)$
$v \approx 5.6$ ft/s

36b. $v = r\frac{\theta}{t}$
$8 = 40\left(\frac{2\pi}{t}\right)$
$t \approx 31$ s

37a. $3 \times 2\pi = 6\pi$ radians 1 minute = 60 seconds
$v = r\frac{\theta}{t}$
$v = 22\frac{1}{2}\left(\frac{6\pi}{60}\right)$
$v \approx 7.1$ ft/s

37b. $v = r\frac{\theta}{t}$
$3.1 = r\left(\frac{6\pi}{60}\right)$
$9.87 \approx r$; about 9.9 ft

37c. $7.1 - 3.1 \approx 4$ ft/s

38a. $35° = 35° \times \frac{\pi}{180°}$
$= \frac{7\pi}{36}$

lighter child: $\omega = \frac{\theta}{t}$
$\omega = \frac{\frac{7\pi}{36}}{\frac{1}{2}}$
$\omega \approx 1.2$ radians/s

heavier child: $\omega = \frac{\theta}{t}$
$\omega = \frac{\frac{7\pi}{36}}{\frac{1}{2}}$
$\omega \approx 1.2$ radians/s

38b. lighter child: $v = r\omega$

$v \approx 9(1.2)$

$v \approx 11.0$ ft/s

heavier child: $v = r\omega$

$v \approx 6(1.2)$

$v \approx 7.3$ ft/s

39a. 3 miles = 190,080 inches

$r = \frac{1}{2}d$ $\qquad s = r\theta$

$r = \frac{1}{2}(30)$ $\qquad 190{,}080 = 15\theta$

$r = 15$ $\qquad 12{,}672 = \theta$

$12{,}672 \times \frac{1 \text{ revolution}}{2\pi} \approx 2017$ revolutions

39b. $\frac{2.75 \text{ revolutions}}{\text{second}} \times \frac{60 \text{ seconds}}{1 \text{ minute}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} \times \frac{2\pi \text{ radians}}{1 \text{ revolution}}$

$= 19{,}800\pi$ radians/hour

$v = r\omega$

$v = 15(19{,}800\pi)$

$v \approx 933{,}053.0181$

$\frac{933{,}053.0181 \text{ inches}}{} \times \frac{1 \text{ mile}}{} \approx 14.7$ mph

40a. Mercury:

$v = r\frac{\theta}{t}$

$v = 2440\left(\frac{2\pi}{1407.6}\right)$

$v \approx 10.9$ km/h

Venus:

$v = r\frac{\theta}{t}$

$v = 6052\left(\frac{2\pi}{5832.5}\right)$

$v \approx 6.5$ km/h

Earth:

$v = r\frac{\theta}{t}$

$v = 6356\left(\frac{2\pi}{23.935}\right)$

$v \approx 1668.5$ km/h

Mars:

$v = r\frac{\theta}{t}$

$v = 3375\left(\frac{2\pi}{24.623}\right)$

$v \approx 861.2$ km/h

40b. The linear velocity of Earth is about twice that of Mars.

41a. $\theta = \theta_m \cos \omega t$

$\theta = \frac{\pi}{4} \cos \pi t$

41b. $\theta = \frac{\pi}{4} \cos \pi t$

$0 = \frac{\pi}{4} \cos \pi t$

$0 = \cos \pi t$

$\pi t = \frac{\pi}{2}$ $\quad$ or $\quad$ $\pi t = \frac{3\pi}{2}$

$t = \frac{1}{2}$ or 0.5 s $\qquad t = \frac{3}{2}$ or 1.5 s

42a. 3960 + 200 = 4160 miles

$C = 2\pi r$ $\qquad t = C \div \text{speed}$

$C = 2\pi(4160)$ $\qquad t \approx 26{,}138.05088 \div 17{,}000$

$C \approx 26138.05088$ $\qquad t \approx 1.537532405$

$\omega = \frac{\theta}{t}$

$\omega \approx \frac{2\pi}{1.54}$

$\omega \approx 4.1$ radians/h

42b. $\omega = \frac{\theta}{t}$ $\qquad t = C \div \text{speed}$

$4 = \frac{2\pi}{t}$ $\qquad \frac{\pi}{2} = 2\pi r \div 17{,}000$

$\frac{\pi}{2} = t$ $\qquad \frac{\pi}{2}(17{,}000) = 2\pi r$

$\frac{\frac{\pi}{2}(17{,}000)}{2\pi} = r$

$4250 = r$

$4250 - 3960 = 290$; about 290 mi

42c. $3960 + 500 = 4460$; $C = 2\pi(4460)$ or 28023.00647

$t = 28{,}023.00647 \div 17{,}000$ or 1.648412145

$\omega = \frac{\theta}{t}$

$\omega \approx \frac{2\pi}{1.65}$

$\omega \approx 3.8$

Its angular velocity is between 3.8 radians/h and 4.1 radians/h.

43a. *B* clockwise; *C* counterclockwise

43b. $v_A = r_A\left(\frac{\theta}{t}\right)_A$

$v_A = 3.0\left(\frac{120}{1}\right)$

$v_A = 360$

The linear velocity of each of the three rollers is the same.

$v_B = r_B\left(\frac{\theta}{t}\right)_B$ $\qquad v_C = r_C\left(\frac{\theta}{t}\right)_C$

$360 = 2.0 \cdot \frac{\theta_B}{1}$ $\qquad 360 = 4.8 \cdot \frac{\theta_C}{1}$

$180 = \theta_B$ $\qquad 75 = \theta_C$

180 rpm $\qquad$ 75 rpm

44. $105° = 105° \times \frac{\pi}{180°}$

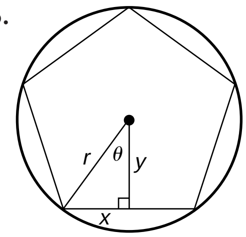
$= \frac{7\pi}{12}$

$A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2}(7.2^2)\left(\frac{7\pi}{12}\right)$

$A \approx 47.5$ cm^2

45.



$r = \frac{1}{2}d$

$r = \frac{1}{2}(7.3)$

$r = 3.65$

$\theta = 360° \div 10$ or $36°$

$\sin \theta = \frac{x}{r}$ $\qquad \cos \theta = \frac{y}{r}$

$\sin 36° = \frac{x}{3.65}$ $\qquad \cos 36° = \frac{y}{3.65}$

$x \approx 2.145416171$ $\qquad y \approx 2.952912029$

$A = \frac{1}{2}bh$

$A \approx \frac{1}{2}(2.15)(2.95)$

$A \approx 3.16761261$

Area of pentagon $\approx 10(3.17)$ or about 31.68 cm^2

46. $35°20'55'' = 35° + 20'\left(\frac{1°}{60'}\right) + 55''\left(\frac{1°}{3600''}\right)$

$\approx 35.349°$

47. $10 + \sqrt{k - 5} = 8$

$\sqrt{k - 5} = -2$

$\sqrt{k - 5} = 4$

$k = 9$

Check: $10 + \sqrt{k - 5} = 8$

$10 + \sqrt{9 - 5} \stackrel{?}{=} 8$

$10 + \sqrt{4} \stackrel{?}{=} 8$

$10 + 2 \stackrel{?}{=} 8$

$12 \neq 8$

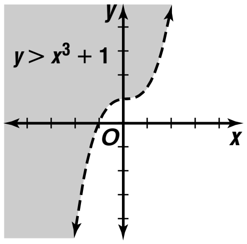
no real solution

48. $(x - (-4))(x - 3\boldsymbol{i})(x - (-3\boldsymbol{i})) = 0$

$(x + 4)(x - 3\boldsymbol{i})(x + 3\boldsymbol{i}) = 0$

$(x + 4)(x^2 + 9) = 0$

$x^3 + 4x^2 + 9x + 36 = 0$

49. 

50. $m = \frac{0 - 5}{-6 - 8}$

$m = \frac{-5}{-14}$ or $\frac{5}{14}$

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{5}{14}(x - (-6))$

$y = \frac{5}{14}x + \frac{15}{7}$

51. $P = 2a + 2b$

$P = 2\left(\frac{3}{4}b\right) + 2b$

$P = \frac{3}{2}b + 2b$

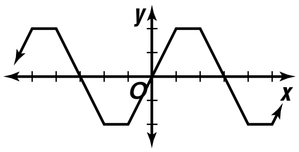
$P = \frac{7}{2}b$

$\frac{2P}{7} = b$ The correct choice is D.

6-3 Graphing Sine and Cosine Functions

Page 363 Check for Understanding

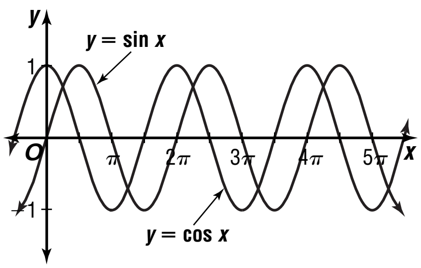
1. Sample answer:



period: 6

2. Sample answers: $-\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}$

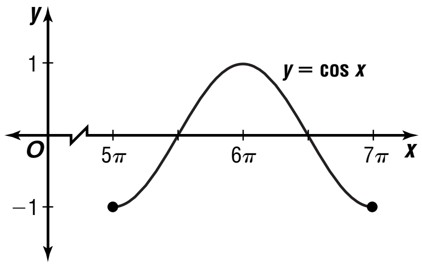
3. $\cos x = \cos(x + 2\pi)$

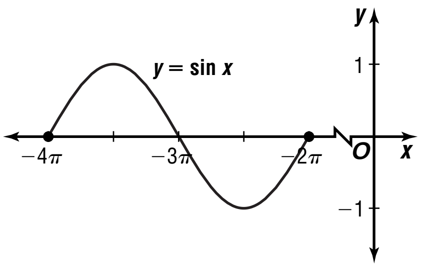
4. 

Both functions are periodic functions with the period of 2π. The domain of both functions is the set of real numbers, and the range of both functions is the set of real numbers between -1 and 1, inclusive. The x-intercepts of the sine function are located at πn, but the x-intercepts of the cosine function are located at $\frac{\pi}{2} + \pi n$, where n is an integer. The y-intercept of the sine function is 0, but the y-intercept of the cosine function is 1. The maximum value of the sine function occurs when $x = \frac{\pi}{2} + 2\pi n$ and its minimum value occurs when $x = \frac{3\pi}{2} + 2\pi n$, where n is an integer. The maximum value of the cosine function occurs when $x = \pi n$, where n is an even integer, and its minimum value occurs when $x = \pi n$, where n is an odd integer.

5. yes; 4 **6.** 0 **7.** 1

8. $\frac{3\pi}{2} + 2\pi n$, where n is an integer

9. 

10. 

11. Neither; the period is not 2π.

12. April (month 4):

$y = 49 + 28 \sin\left[\frac{\pi}{6}(t - 4)\right]$

$y = 49 + 28 \sin\left[\frac{\pi}{6}(4 - 4)\right]$

$y = 49$

October (month 10):

$y = 49 + 28 \sin\left[\frac{\pi}{6}(t - 4)\right]$

$y = 49 + 28 \sin\left[\frac{\pi}{6}(10 - 4)\right]$

$y = 49$

The average temperatures are the same.

Pages 363–366 Exercises

13. yes; 6 **14.** no **15.** yes; 20 **16.** no

17. no **18.** no **19.** 1 **20.** 0

21. 0 **22.** 1 **23.** -1 **24.** -1

25. $\sin \pi + \cos \pi = 0 + (-1)$
$= -1$

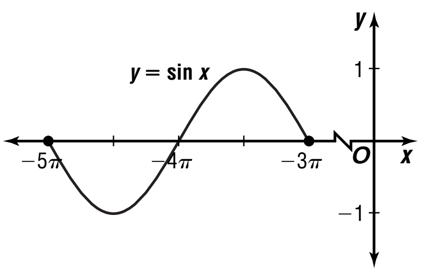
26. $\sin 2\pi - \cos 2\pi = 0 - 1$
$= -1$

27. $\pi + 2\pi n$, where n is an integer

28. $\frac{\pi}{2} + 2\pi n$, where n is an integer

29. $\frac{\pi}{2} + \pi n$, where n is an integer

30. $\theta + 2\pi n$, where n is an integer

31. 

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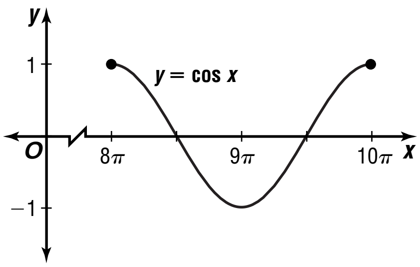

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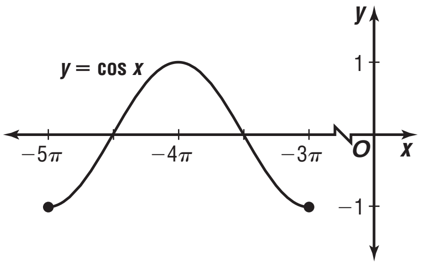

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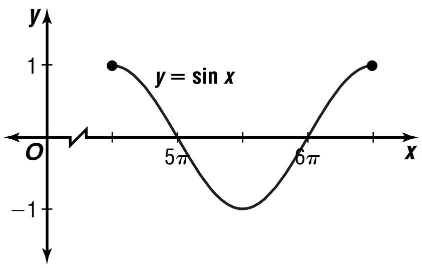

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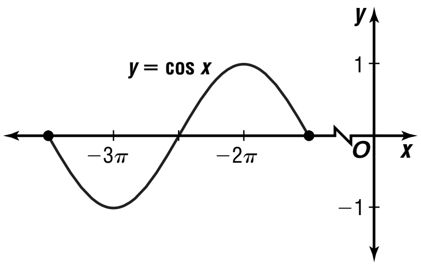
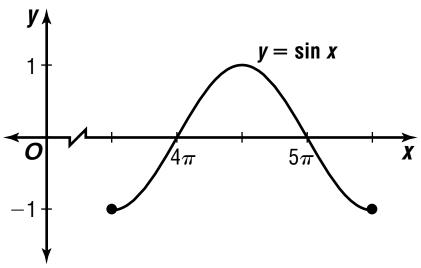


36.


37. $y = \cos x$; the maximum value of 1 occurs when $x = 4\pi$, the minimum value of -1 occurs when $x = 5\pi$, and the x-intercepts are $\frac{7\pi}{2}$, $\frac{9\pi}{2}$, and $\frac{11\pi}{2}$.

38. Neither; the graph does not cross the x-axis.

39. $y = \sin x$; the maximum value of 1 occurs when $x = -\frac{11\pi}{2}$, the minimum value of -1 occurs when $x = -\frac{13\pi}{2}$, and the x-intercepts are -7π, -6π, and -5π.

40. Sample answer: a shift of $\frac{\pi}{2}$ to the left

41. $x = \frac{\pi}{2} + \pi n$, where n is an integer

42. $x = \pi n$, where n is an integer

43a. $\csc \theta = \frac{1}{\sin \theta}$
$1 = \frac{1}{\sin \theta}$
$\sin \theta = 1$
$\frac{\pi}{2} + 2\pi n$, where n is an integer

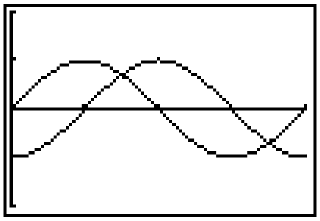
43b. $\csc \theta = \frac{1}{\sin \theta}$
$-1 = \frac{1}{\sin \theta}$
$\sin \theta = -1$
$\frac{3\pi}{2} + 2\pi n$, where n is an integer

43c. $\csc \theta$ is undefined when $\sin \theta = 0$.
πn, where n is an integer

44a. $\sec \theta = \frac{1}{\cos \theta}$
$1 = \frac{1}{\cos \theta}$
$\cos \theta = 1$
$2\pi n$, where n is an integer

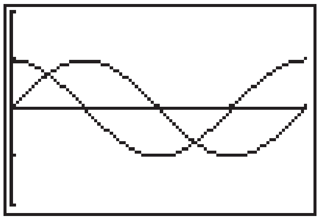
44b. $\sec \theta = \frac{1}{\cos \theta}$
$-1 = \frac{1}{\cos \theta}$
$\cos \theta = -1$
$\pi + 2\pi n$, where n is an integer

44c. $\sec \theta$ is undefined when $\cos \theta = 0$.
$\frac{\pi}{2} + \pi n$, where n is an integer

45.


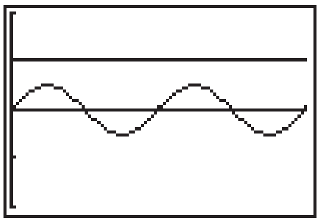
$[0, 2\pi]$ sc1:$\frac{\pi}{2}$ by $[-2, 2]$ sc1:1

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

46.


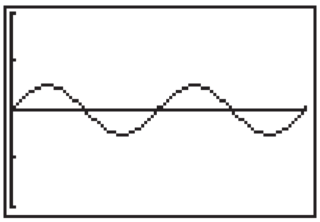
$[0, 2\pi]$ sc1:$\frac{\pi}{2}$ by $[-2, 2]$ sc1:1

$0 \le x \le \frac{\pi}{4}, \frac{5\pi}{4} \le x \le 2\pi$

47.


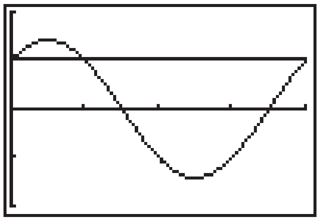
$[0, 2\pi]$ sc1:$\frac{\pi}{2}$ by $[-2, 2]$ sc1:1

none

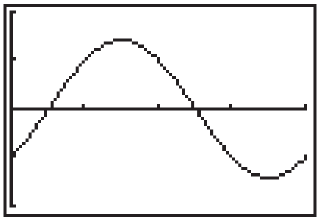
48.


$[0, 2\pi]$ sc1:$\frac{\pi}{2}$ by $[-2, 2]$ sc1:1

$x = 0, \frac{\pi}{2} \le x \le \pi, \frac{3\pi}{2} \le x \le 2\pi$

49. 

$[0, 2\pi]$ sc1: $\frac{\pi}{2}$ by $[-2, 2]$ sc1:1
$x = 0, \frac{\pi}{2}, 2\pi$

50. 

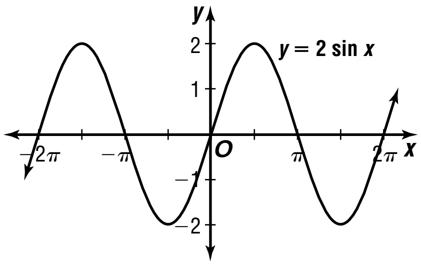
$[0, 2\pi]$ sc1: $\frac{\pi}{2}$ by $[-2, 2]$ sc1:1
$x = \frac{\pi}{4}, \frac{5\pi}{4}$

51a. July (month 7):
$y = 43 + 31 \sin\left[\frac{\pi}{6}(t - 4)\right]$
$y = 43 + 31 \sin\left[\frac{\pi}{6}(7 - 4)\right]$
$y = 74$
January (month 1):
$y = 43 + 31 \sin\left[\frac{\pi}{6}(t - 4)\right]$
$y = 43 + 31 \sin\left[\frac{\pi}{6}(1 - 4)\right]$
$y = 12$
$74 - 12 = 62$; it is twice the coefficient.

51b. Using answers from 51a., $74 + 12 = 86$; it is twice the constant term.

52a. πn, where n is an integer

52b. 2 **52c.** -2 **52d.** 2π

52e. 

52f. It expands the graph vertically.

53a. $P = 100 + 20 \sin 2\pi t$
$P = 100 + 20 \sin 2\pi(0)$ or 100
$P = 100 + 20 \sin 2\pi(0.25)$ or 120
$P = 100 + 20 \sin 2\pi(0.5)$ or 100
$P = 100 + 20 \sin 2\pi(0.75)$ or 80
$P = 100 + 20 \sin 2\pi(1)$ or 100

53b. 0.25 s **53c.** 0.75 s

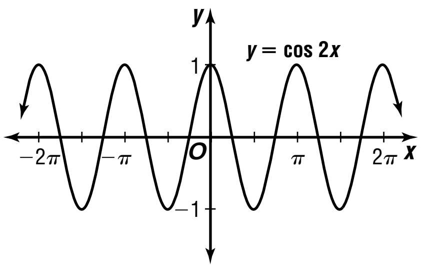
54a. $v = 3.5 \cos\left(t\sqrt{\frac{k}{m}}\right)$
$v = 3.5 \cos\left(0.9\sqrt{\frac{19.6}{1.99}}\right)$
$v \approx -3.3$ cm
$v = 3.5 \cos\left(t\sqrt{\frac{k}{m}}\right)$
$v = 3.5 \cos\left(1.7\sqrt{\frac{19.6}{1.99}}\right)$
$v \approx 2.0$ cm

54b. $v = 3.5 \cos\left(t\sqrt{\frac{k}{m}}\right)$
$0 = 3.5 \cos\left(t\sqrt{\frac{19.6}{1.99}}\right)$
$0 = \cos\left(t\sqrt{\frac{19.6}{1.99}}\right)$
$\cos^{-1} 0 = t\sqrt{\frac{19.6}{1.99}}$
$1.570796327 \approx t\sqrt{\frac{19.6}{1.99}}$
$0.5005164776 \approx t$; about 0.5 s

54c. $v = 3.5 \cos\left(t\sqrt{\frac{k}{m}}\right)$
$3.5 = 3.5 \cos\left(t\sqrt{\frac{19.6}{1.99}}\right)$
$1 = \cos\left(t\sqrt{\frac{19.6}{1.99}}\right)$
$\cos^{-1} 1 = t\sqrt{\frac{19.6}{1.99}}$
$2\pi = t\sqrt{\frac{19.6}{1.99}}$
$2.00206591 \approx t$; about 2.0 s

55a. $\frac{\pi}{4} + \frac{\pi n}{2}$, where n is an integer

55b. 1 **55c.** -1 **55d.** π

55e. 

56a. $P = 500 + 200 \sin [0.4(t - 2)]$
$P = 500 + 200 \sin [0.4(0 - 2)]$ or about 357 pumas
$D = 1500 + 400 \sin (0.4t)$
$D = 1500 + 400 \sin (0.4(0))$ or 1500 deer

56b. $P = 500 + 200 \sin [0.4(t - 2)]$
$P = 500 + 200 \sin [0.4(10 - 2)]$ or about 488 pumas
$D = 1500 + 400 \sin (0.4t)$
$D = 1500 + 400 \sin (0.4(10))$ or about 1197 deer

56c. $P = 500 + 200 \sin [0.4(t - 2)]$
$P = 500 + 200 \sin [0.4(25 - 2)]$ or about 545 pumas
$D = 1500 + 400 \sin (0.4t)$
$D = 1500 + 400 \sin (0.4(25))$ or about 1282 deer

57. $\frac{500 \text{ revolutions}}{1 \text{ minute}} \times \frac{1 \text{ minute}}{60 \text{ seconds}} \times \frac{2\pi \text{ radians}}{1 \text{ revolution}} \approx 52.4$ radians per second

58. $-1.5 = -1.5 \times \frac{180°}{\pi}$
$\approx -85.9°$

59. 45°, 135°

60. $\frac{2}{x+2} = \frac{x}{2-x} + \frac{x^2+4}{x^2-4}$

$-1(x+2)(x-2)\left(\frac{2}{x+2}\right) = -1(x+2)(x-2)\left(\frac{x}{2-x}\right) + (-1)(x+2)(x-2)\left(\frac{x^2+4}{x^2-4}\right)$

$-1(x-2)(2) = (x+2)(x) + (-1)(x^2+4)$

$-2x + 4 = x^2 + 2x - x^2 - 4$

$x = 2$

But, $x \neq 2$, so there is no solution.

61. 1 positive real zero

$f(-x) = -2x^3 + 3x^2 + 11x - 6$

2 or 0 negative real zeros

2	2	3	−11	−6
		4	14	6
	2	7	3	0

$2x^2 + 7x + 3 = 0$

$(2x+1)(x+3) = 0$

$2x + 1 = 0$ or $x + 3 = 0$

$x = -\frac{1}{2}$ $\quad x = -3$

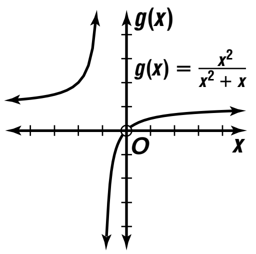
$-3, -\frac{1}{2}, 2$

62.

1	1	2	−9	18
		1	3	−6
	1	3	−6	12

12; no

63.



vertical:

$x^2 + x = 0$

$x(x+1) = 0$

$x = 0$ or $x + 1 = 0$

$x = -1$

horizontal:

$y = 1$

64. reflected over the x-axis, expanded vertically by a factor of 3

65. $\begin{vmatrix} -2 & 4 & -1 \\ 1 & -1 & 0 \\ -3 & 4 & 5 \end{vmatrix}$

$= -2\begin{vmatrix} -1 & 0 \\ 4 & 5 \end{vmatrix} - 4\begin{vmatrix} 1 & 0 \\ -3 & 5 \end{vmatrix} + (-1)\begin{vmatrix} 1 & -1 \\ -3 & 4 \end{vmatrix}$

$= -2(-5) - 4(5) - 1(1)$

$= -11$

66. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & 2 & 1 \\ 2 & -4 & 6 \end{bmatrix}$

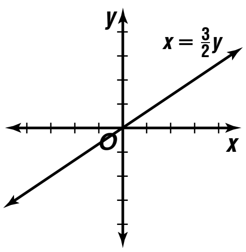
$= \begin{bmatrix} -1(3)+0(2) & -1(2)+0(-4) & -1(1)+0(6) \\ 0(3)+1(2) & 0(2)+1(-4) & 0(1)+1(6) \end{bmatrix}$

$= \begin{bmatrix} -3 & -2 & -1 \\ 2 & -4 & 6 \end{bmatrix}$

$A'(-3, 2), B'(-2, -4), C'(-1, 6)$

67. $x = \frac{3}{2}y$

$y = \frac{2}{3}x$



68. Perimeter of square $RSVW$

$= RS + SV + VW + WR$

$= 5 + 5 + 5 + 5$ or 20

Perimeter of rectangle $RTUW$

$= RT + TU + UW + WR$

$= (5 + 2) + 5 + (5 + 2) + 5$

$= 24$

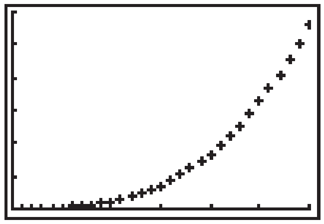
$24 - 20 = 4$

The correct choice is B.

Page 367 History of Mathematics

1.

n	$n^2 + n^3$
1	2
2	12
3	36
4	80
5	150
6	252
7	392
8	576
9	810
10	1100
11	1452
12	1872
13	2366
14	2940
15	3600
16	4352
17	5202
18	6156
19	7220
20	8400
21	9702
22	11,132
23	12,696
24	14,400
25	16,250
26	18,252
27	20,412
28	22,736
29	25,230
30	27,900



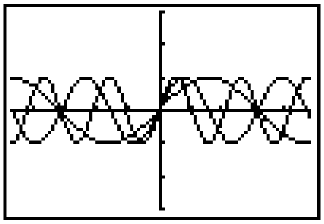
[0, 30] sc1:5 by [0, 30,000] sc1:5000

The graph is not a straight line. It curves upward, increasing more rapidly as the value of n increases.

2. See students' work.

6-4 Amplitude and Period of Sine and Cosine Functions

Page 369 Graphing Calculator Exploration

1. 

2. The graph is shrunk horizontally.
3. The graph of $f(x) = \sin kx$ for $k < 0$ is the graph of $f(x) = \sin |k|x$ reflected over the y-axis.

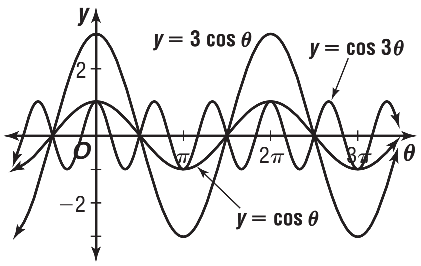
Pages 372–373 Check for Understanding

1. Sample answer: $y = 5 \sin 2\theta$
2. The graphs are a reflection of each other over the θ-axis.
3. A: period $= \frac{2\pi}{2}$ or π

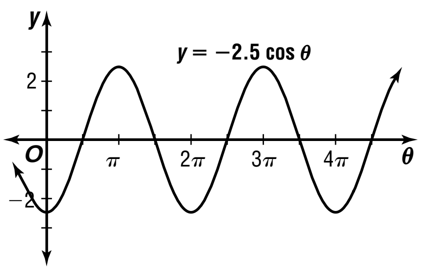
   B: period $= \frac{2\pi}{5}$

   C: period $= \frac{2\pi}{\frac{1}{2}}$ or 4π

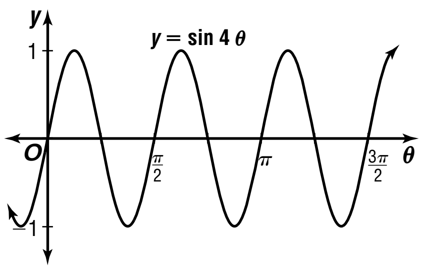
   D: period $= 2\pi$

   C has the greatest period.
4. Period and frequency are reciprocals of each other.
5. 

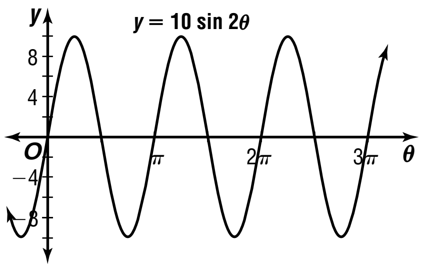
   All three graphs are periodic and curve above and below the x-axis. The amplitude of $y = 3 \cos \theta$ is 3, while the amplitude of $y = \cos \theta$ and $y = \cos 3\theta$ is 1. The period of $y = \cos 3\theta$ is $\frac{2\pi}{3}$, while the period of $y = \cos \theta$ and $y = 3 \cos \theta$ is 2π.
6. $|-2.5| = 2.5$



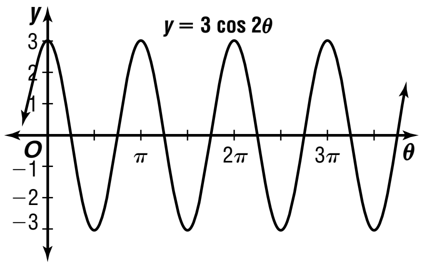
7. $\frac{2\pi}{4} = \frac{\pi}{2}$



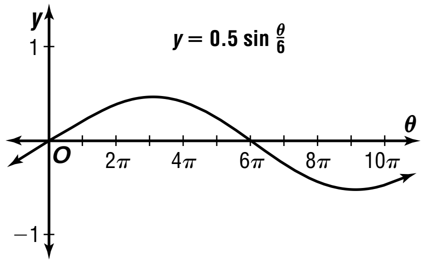
8. $|10| = 10; \frac{2\pi}{2} = \pi$



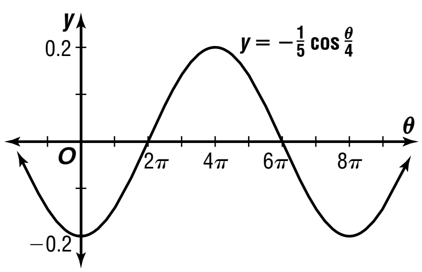
9. $|3| = 3; \frac{2\pi}{2} = \pi$



10. $|0.5| = 0.5; \frac{2\pi}{\frac{1}{6}} = 12\pi$



11. $\left|-\frac{1}{5}\right| = \frac{1}{5}; \frac{2\pi}{\frac{1}{4}} = 8\pi$



12. $|A| = 0.8$ $\qquad \frac{2\pi}{k} = \pi$

    $A = \pm 0.8$ $\qquad k = \frac{2\pi}{\pi}$ or 2

    $y = \pm 0.8 \sin 2\theta$

13. $|A| = 7$ $\qquad \frac{2\pi}{k} = \frac{\pi}{3}$

$A = \pm 7$ $\qquad k = \frac{2\pi}{\frac{\pi}{3}}$ or 6

$y = \pm 7 \sin 6\theta$

14. $|A| = 1.5$ $\qquad \frac{2\pi}{k} = 5\pi$

$A = \pm 1.5$ $\qquad k = \frac{2\pi}{5\pi}$ or $\frac{2}{5}$

$y = \pm 1.5 \cos \frac{2}{5}\theta$

15. $|A| = \frac{3}{4}$ $\qquad \frac{2\pi}{k} = 6$

$A = \pm\frac{3}{4}$ $\qquad k = \frac{2\pi}{6}$ or $\frac{\pi}{3}$

$y = \pm\frac{3}{4} \cos \frac{\pi}{3}\theta$

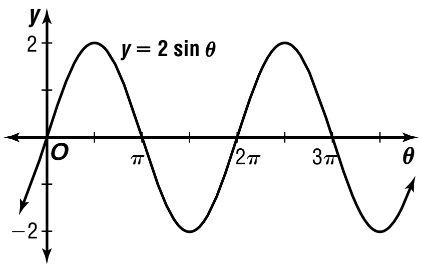
16. $|A| = 0.25$ $\qquad \frac{2\pi}{k} = \frac{1}{294}$

$A = \pm 0.25$ $\qquad k = 588\pi$

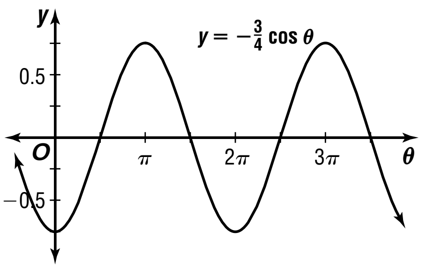
$y = \pm 0.25 \sin (588\pi \times t)$

Pages 373–377 Exercises

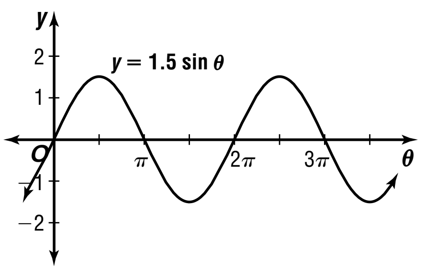
17. $|2| = 2$



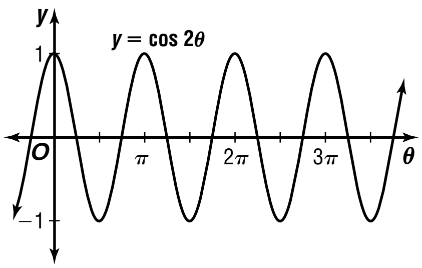
18. $\left|-\frac{3}{4}\right| = \frac{3}{4}$



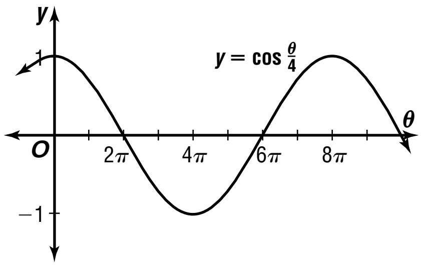
19. $|1.5| = 1.5$



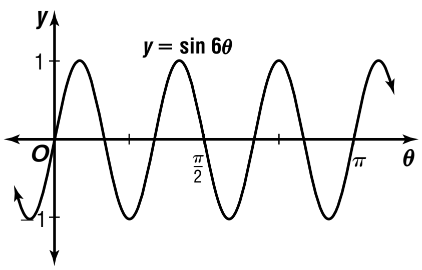
20. $\frac{2\pi}{2} = \pi$



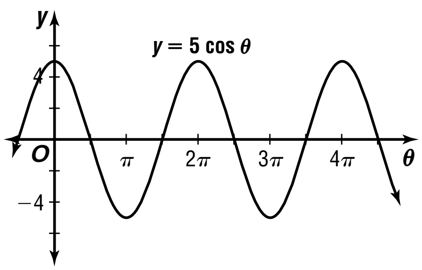
21. $\frac{2\pi}{\frac{1}{4}} = 8\pi$



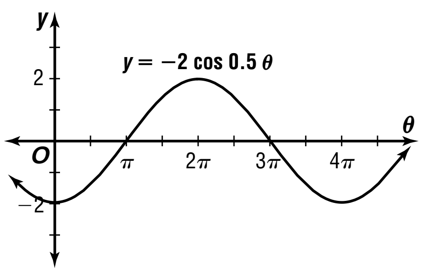
22. $\frac{2\pi}{6} = \frac{\pi}{3}$



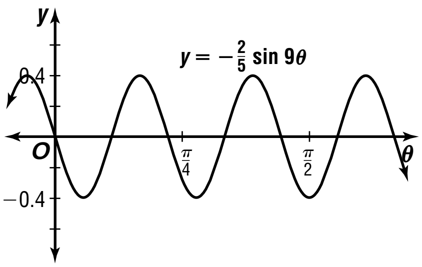
23. $|5| = 5$; $\frac{2\pi}{1} = 2\pi$



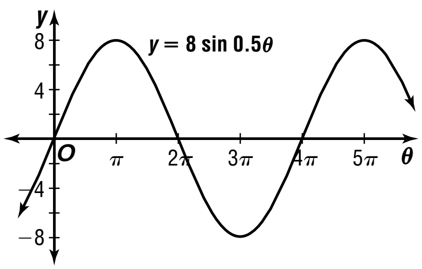
24. $|-2| = 2$; $\frac{2\pi}{0.5} = 4\pi$



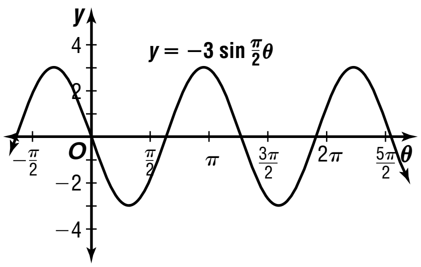
25. $\left|-\frac{2}{5}\right| = \frac{2}{5}; \frac{2\pi}{9}$



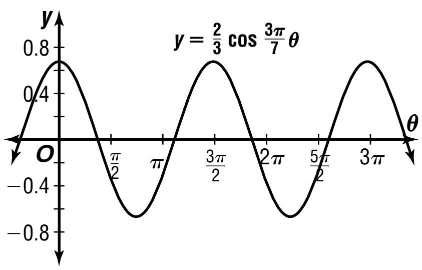
26. $|8| = 8; \frac{2\pi}{0.5} = 4\pi$



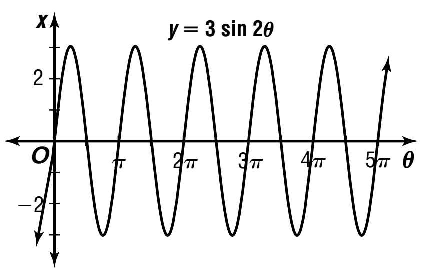
27. $|-3| = 3; \frac{2\pi}{\frac{\pi}{2}} = 4$



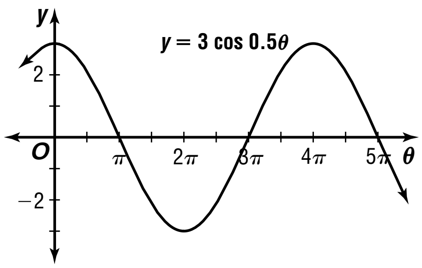
28. $\left|\frac{2}{3}\right| = \frac{2}{3}; \frac{2\pi}{\frac{3\pi}{7}} = \frac{14}{3}$



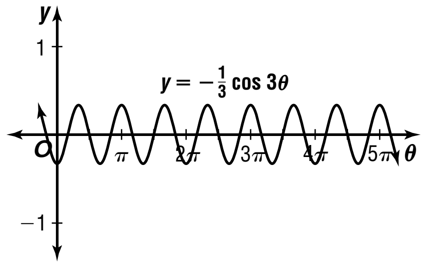
29. $|3| = 3; \frac{2\pi}{2} = \pi$



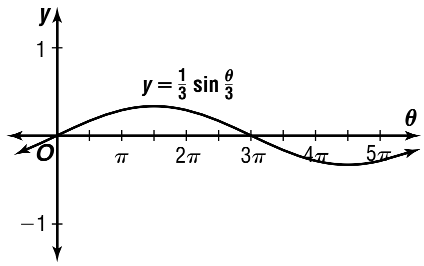
30. $|3| = 3; \frac{2\pi}{0.5} = 4\pi$



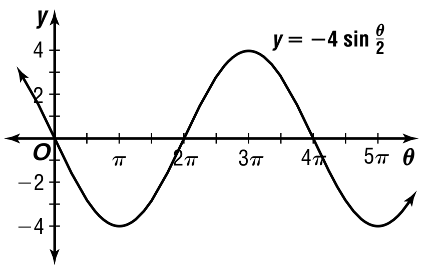
31. $\left|-\frac{1}{3}\right| = \frac{1}{3}; \frac{2\pi}{3}$



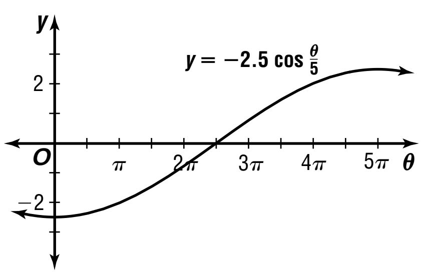
32. $\left|\frac{1}{3}\right| = \frac{1}{3}; \frac{2\pi}{\frac{1}{3}} = 6\pi$



33. $|-4| = 4; \frac{2\pi}{\frac{1}{2}} = 4\pi$



34. $|-2.5| = 2.5; \frac{2\pi}{\frac{1}{5}} = 10\pi$



35. $|0.5| = 0.5; \frac{2\pi}{698\pi} = \frac{1}{349}$

36. $|A| = 0.4$ $\quad \frac{2\pi}{k} = 10\pi$

$A = \pm 0.4$ $\quad k = \frac{1}{5}$

$y = \pm 0.4 \sin \frac{\theta}{5}$

37. $|A| = 35.7$ $\quad \frac{2\pi}{k} = \frac{\pi}{4}$

$A = \pm 35.7$ $\quad k = 8$

$y = \pm 35.7 \sin 8\theta$

38. $|A| = \frac{1}{4}$ $\quad \frac{2\pi}{k} = \frac{\pi}{3}$

$A = \pm \frac{1}{4}$ $\quad k = 6$

$y = \pm \frac{1}{4} \sin 6\theta$

39. $|A| = 0.34$ $\quad \frac{2\pi}{k} = 0.75\pi$

$A = \pm 0.34$ $\quad k = \frac{8}{3}$

$y = \pm 0.34 \sin \frac{8}{3}\theta$

40. $|A| = 4.5$ $\quad \frac{2\pi}{k} = \frac{5\pi}{4}$

$A = \pm 4.5$ $\quad k = \frac{8}{5}$

$y = \pm 4.5 \sin \frac{8}{5}\theta$

41. $|A| = 16$ $\quad \frac{2\pi}{k} = 30$

$A = \pm 16$ $\quad k = \frac{\pi}{15}$

$y = \pm 16 \sin \frac{\pi}{15}\theta$

42. $|A| = 5$ $\quad \frac{2\pi}{k} = 2\pi$

$A = \pm 5$ $\quad k = 1$

$y = \pm 5 \cos \theta$

43. $|A| = \frac{5}{8}$ $\quad \frac{2\pi}{k} = \frac{\pi}{7}$

$A = \pm \frac{5}{8}$ $\quad k = 14$

$y = \pm \frac{5}{8} \cos 14\theta$

44. $|A| = 7.5$ $\quad \frac{2\pi}{k} = 6\pi$

$A = \pm 7.5$ $\quad k = \frac{1}{3}$

$y = \pm 7.5 \cos \frac{\theta}{3}$

45. $|A| = 0.5$ $\quad \frac{2\pi}{k} = 0.3\pi$

$A = \pm 0.5$ $\quad k = \frac{20}{3}$

$y = \pm 0.5 \cos \frac{20}{3}\theta$

46. $|A| = \frac{2}{5}$ $\quad \frac{2\pi}{k} = \frac{3}{5}\pi$

$A = \pm \frac{2}{5}$ $\quad k = \frac{10}{3}$

$y = \pm \frac{2}{5} \cos \frac{10}{3}\theta$

47. $|A| = 17.9$ $\quad \frac{2\pi}{k} = 16$

$A = \pm 17.9$ $\quad k = \frac{\pi}{8}$

$y = \pm 17.9 \cos \frac{\pi}{8}\theta$

48. $|A| = 1.5$ $\quad \frac{2\pi}{k} = \frac{\pi}{2}$

$A = \pm 1.5$ $\quad k = 4$

$y = \pm 1.5 \sin 4\theta, y = \pm 1.5 \cos 4\theta$

49. cosine curve $\quad A = 2$ $\quad \frac{2\pi}{k} = 4\pi$

$k = \frac{1}{2}$

$y = 2 \cos \frac{\theta}{2}$

50. sine curve $\quad A = 0.5$ $\quad \frac{2\pi}{k} = \pi$

$k = 2$

$y = 0.5 \sin 2\theta$

51. cosine curve $\quad A = -3$ $\quad \frac{2\pi}{k} = 2\pi$

$k = 1$

$y = -3 \cos \theta$

52. sine curve $\quad A = -1.5$ $\quad \frac{2\pi}{k} = 4\pi$

$k = \frac{1}{2}$

$y = -1.5 \sin \frac{\theta}{2}$

53. $|A| = 3.8$ $\quad \frac{2\pi}{k} = \frac{1}{120}$

$A = \pm 3.8$ $\quad k = 240\pi$

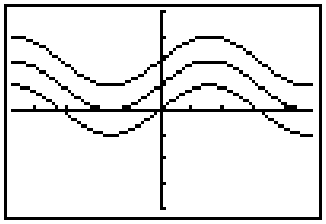
$y = \pm 3.8 \sin(240\pi \times t)$

54. $|A| = 15$ $\quad \frac{2\pi}{k} = \frac{1}{36}$

$A = \pm 15$ $\quad k = 72\pi$

$y = \pm 15 \cos (72\pi \times t)$

55.



All the graphs have the same shape, but have been translated vertically.

56a. $|A| = \left|\frac{A - (-A)}{2}\right|$ $\quad \frac{2\pi}{k} = 8$

$k = \frac{\pi}{4}$

$|A| = \left|\frac{3}{2}\right|$

$A = \pm 1.5$; down first, so $A = -1.5$

$y = -1.5 \sin \frac{\pi}{4}t$

56b. $y = -1.5 \sin \frac{\pi}{4}t$

$y = -1.5 \sin \frac{\pi}{4}(3)$

$y \approx -1.1$ ft

56c. $y = -1.5 \sin \frac{\pi}{4}t$

$y = -1.5 \sin \frac{\pi}{4}(12)$

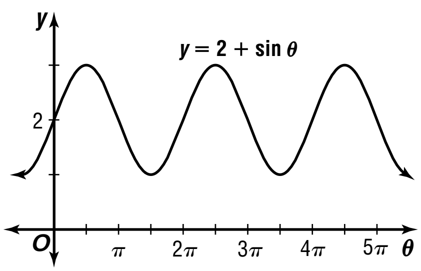
$y = 0$ ft

57a. Maximum value of $\sin \theta = 1$.
Maximum value of $2 + \sin \theta = 2 + 1$ or 3

57b. Minimum value of $\sin \theta = -1$
Minimum value of $2 + \sin \theta = 2 + (-1)$ or 1

57c. $\frac{2\pi}{1} = 2\pi$

37d.



58a. $|A| = 0.2$ $\quad \frac{2\pi}{k} = \frac{1}{262}$

$A = \pm 0.2$ $\quad k = 524\pi$

$y = \pm 0.2 \sin (524\pi \times t)$

58b. $|A| = \frac{1}{2}(0.2)$ $\quad \frac{2\pi}{k} = \frac{1}{524}$

$A = \pm 0.1$ $\quad k = 1048\pi$

$y = \pm 0.1 \sin (1048\pi \times t)$

58c. $|A| = 2(0.2)$ $\quad \frac{2\pi}{k} = \frac{1}{131}$

$A = \pm 0.4$ $\quad k = 262\pi$

$y = \pm 0.4 \sin (262\pi \times t)$

59a. $y = A \cos \left(t\sqrt{\frac{g}{\ell}}\right)$

$y = 1.5 \cos \left(t\sqrt{\frac{9.8}{6}}\right)$

59b. $y = 1.5 \cos \left(t\sqrt{\frac{9.8}{6}}\right)$

$y = 1.5 \cos \left(4\sqrt{\frac{9.8}{6}}\right)$

$y \approx 0.6$

about 0.6 m to the right

59c. $y = 1.5 \cos \left(t\sqrt{\frac{9.8}{6}}\right)$

$y = 1.5 \cos \left(7.9\sqrt{\frac{9.8}{6}}\right)$

$y \approx -1.2$

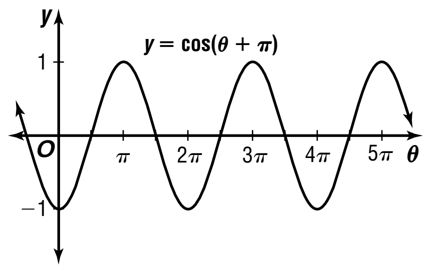
about 1.2 m to the left

60a. $\frac{\pi}{2} + \pi n$, where n is an integer

60b. -1

60c. $\frac{2\pi}{1} = 2\pi$

60d.



61a. $y = 1.5 \cos \left(t\sqrt{\frac{k}{m}}\right)$ $\quad \frac{2\pi}{k} \approx 6.8$

$y = 1.5 \cos \left(t\sqrt{\frac{18.5}{0.4}}\right)$ $\quad k \approx 0.9$ s/cycle

$y \approx 1.5 \cos 6.8t$ $\quad$ frequency: $\frac{1}{0.9} \approx 1.1$ hertz

61b. $y = 1.5 \cos \left(t\sqrt{\frac{k}{m}}\right)$ $\quad \frac{2\pi}{k} \approx 5.6$

$y = 1.5 \cos \left(t\sqrt{\frac{18.5}{0.6}}\right)$ $\quad k \approx 1.1$ s/cycle

$y \approx 1.5 \cos 5.6t$ $\quad$ frequency: $\frac{1}{1.1} \approx 0.9$ hertz

61c. $y = 1.5 \cos \left(t\sqrt{\frac{k}{m}}\right)$ $\quad \frac{2\pi}{k} \approx 4.8$

$y = 1.5 \cos \left(t\sqrt{\frac{18.5}{0.8}}\right)$ $\quad k \approx 1.3$ s/cycle

$y \approx 1.5 \cos 4.8t$ $\quad$ frequency: $\frac{1}{1.3} \approx 0.8$ hertz

61d. It increases. **61e.** It decreases.

62. 0

63. $84 \times 2\pi = 168\pi$ radians

$\omega = \frac{\theta}{t}$

$\omega = \frac{168\pi}{6}$

$\omega \approx 88.0$ radians/s

64. $73° = 73° \times \frac{\pi}{180°}$ $\quad s = r\theta$

$= \frac{73\pi}{180}$ $\quad s = 9\left(\frac{73\pi}{180}\right)$

$s \approx 11.5$ in.

65. $a^2 + b^2 = c^2$ $\quad \tan A = \frac{a}{b}$

$15.1^2 + 19.5^2 = c^2$ $\quad \tan A = \frac{15.1}{19.5}$

$24.66292764 \approx c$ $\quad A = \tan^{-1} \frac{15.1}{19.5}$

$A \approx 37.75273111$

$B = 180° - (90° + 37.8°)$ or $52.2°$

$c = 24.7, A = 37.8°, B = 52.2°$

66. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$4.1 = 2\pi\sqrt{\frac{\ell}{9.8}}$

$0.6525352667 \approx \sqrt{\frac{\ell}{9.8}}$

$0.458022743 \approx \frac{\ell}{9.8}$

$4.17 \approx \ell$; about 4.17 m

67. $b^2 - 4ac = 5^2 - 4(3)(10)$

$= -95$

2 imaginary roots

68a. Let x = the number of Model 28 cards and let y = the number of Model 74 cards.

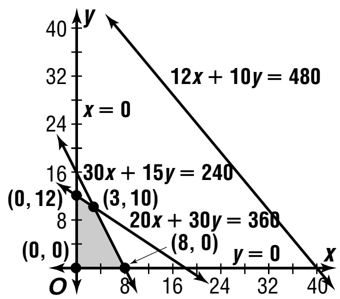
$30x + 15y \leq 240$

$20x + 30y \leq 360$

$12x + 10y \leq 480$

$x \geq 0$

$y \geq 0$



$P(x, y) = 100x + 60y$

$P(0, 0) = 100(0) + 60(0)$ or 0

$P(0, 12) = 100(0) + 60(12)$ or 720

$P(3, 10) = 100(3) + 60(10)$ or 900

$P(0, 8) = 100(0) + 60(8)$ or 480

3 of Model 28, 10 of Model 74

68b. $900

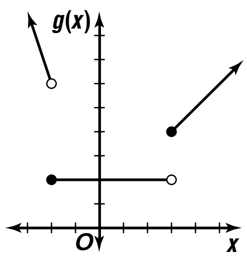
69. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 & 3 & -3 \\ -1 & -1 & -4 & -2 \end{bmatrix}$

$= \begin{bmatrix} 1(-2) + 0(-1) & 1(1) + 0(-1) & 1(3) + 0(-4) & 1(-3) + 0(-2) \\ 0(-2) + (-1)(-1) & 0(1) + (-1)(-1) & 0(3) + (-1)(-4) & 0(-3) + (-1)(-2) \end{bmatrix}$

$= \begin{bmatrix} -2 & 1 & 3 & -3 \\ 1 & 1 & 4 & 2 \end{bmatrix}$

$(-2, 1), (1, 1), (3, 4), (-3, 2)$

70.



71. $y = 14.7x + 140.1$
$y = 14.7(20) + 140.1$
$y = \$434.10$

72.

x	x^2	y
-4	$(-4)^2$	16
-3	$(-3)^2$	9
-2	$(-2)^2$	4

$\{(-4, 16), (-3, 9), (-2, 4)\}$; yes

73. $A = s^2$
$100 = s^2$
$10 = s$

radius $= \frac{1}{2}(10)$ or 5
$A = \pi r^2$
$A = \pi(5)^2$ or 25π
$4(25\pi) = 100\pi$

The correct choice is C.

Page 377 Mid-Chapter Quiz

1. $\frac{5\pi}{6} = \frac{5\pi}{6} \times \frac{180°}{\pi}$
$= 150°$

2. $r = \frac{1}{2}d$
$r = \frac{1}{2}(0.5)$ or 0.25

$s = r\theta$
$s = 0.25\left(\frac{5\pi}{3}\right)$
$s \approx 1.3$ m

3. $A = \frac{1}{2}r^2\theta$
$A = \frac{1}{2}(8^2)\left(\frac{2\pi}{5}\right)$
$A \approx 40.2$ ft^2

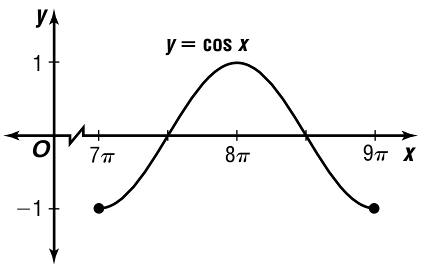
4. $7.8 \times 2\pi = 15.6\pi$ or about 49.0 radians

5. $8.6 \times 2\pi = 17.2\pi$
$\omega = \frac{\theta}{t}$
$\omega = \frac{17.2\pi}{7}$
$\omega \approx 7.7$ radians/s

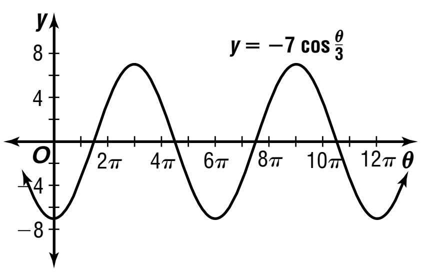
6. $v = r\omega$
$v = 3(8\pi)$
$v \approx 75.4$ meters/s

7. 1

8.



9. $|-7| = 7$; $\frac{2\pi}{\frac{1}{3}} = 6\pi$



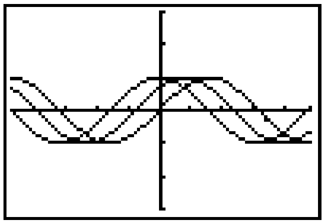
10. $|A| = 5$
$A = \pm 5$
$y = \pm 5 \sin 6\theta$

$\frac{2\pi}{k} = \frac{\pi}{3}$
$k = 6$

6-5 Translations of Sine and Cosine Functions

Page 378 Graphing Calculator Exploration

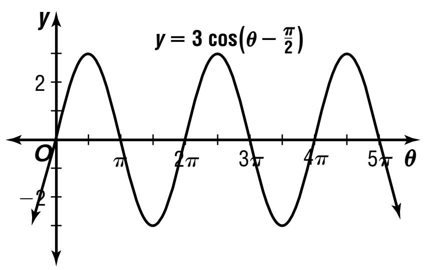
1.



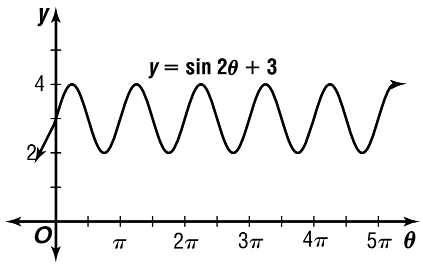
2. The graph shifts farther to the left.
3. The graph shifts farther to the right.

Page 383 Check for Understanding

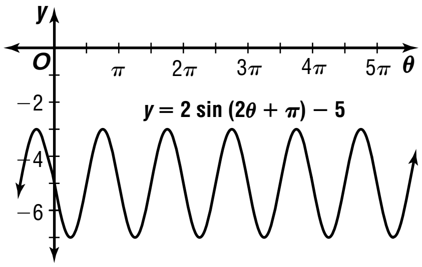
1. Both graphs are the sine curve. The graph of $y = \sin x + 1$ has a vertical shift of 1 unit upward, while the graph of $y = \sin(x + 1)$ has a horizontal shift of 1 unit to the left.
2. sine function
3a. increase $|A|$
3b. decrease h
3c. increase $|k|$
3d. increase c
4. Graph $y = \sin x$ and $y = \cos x$, and find the sum of their ordinates.
5. Jamal; $-\frac{c}{k} = -\frac{-\frac{\pi}{2}}{\frac{\pi}{6}}$ or 3
6. $\frac{\pi}{2}$



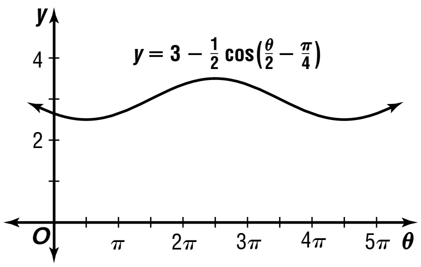
7. 3; $y = 3$
$A = 1$;
$\frac{2\pi}{2} = \pi$



8. $|2| = 2; \frac{2\pi}{2} = \pi; -\frac{\pi}{2}; -5$



9. $\left|-\frac{1}{2}\right| = \frac{1}{2}; \frac{2\pi}{\frac{1}{2}} = 4\pi; -\frac{-\frac{\pi}{4}}{\frac{1}{2}} = \frac{\pi}{2}; 3$



10. $|A| = 20 \qquad \frac{2\pi}{k} = 1 \qquad c = 0$

$A = \pm 20 \qquad k = 2\pi \qquad h = 100$

$y = \pm 20 \sin 2\pi\theta + 100$

11. $|A| = 0.6 \qquad \frac{2\pi}{k} = 12.4$

$A = \pm 0.6 \qquad k = \frac{\pi}{6.2}$

$-\frac{c}{k} = -2.13 \qquad h = 7$

$-\frac{c}{\frac{\pi}{6.2}} = -2.13$

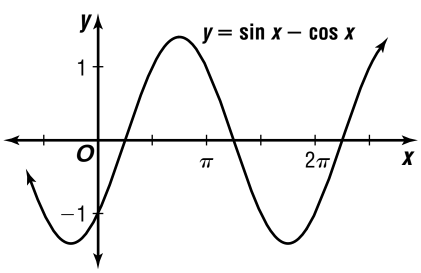
$c = \frac{2.13\pi}{6.2}$

$y = \pm 0.6 \cos\left(\frac{\pi}{6.2}\theta + \frac{2.13\pi}{6.2}\right)$

$+ 7$

12.

x	$\sin x - \cos x$	y
0	$\sin 0 - \cos 0$	−1
$\frac{\pi}{2}$	$\sin \frac{\pi}{2} - \cos \frac{\pi}{2}$	1
π	$\sin \pi - \cos \pi$	1
$\frac{3\pi}{2}$	$\sin \frac{3\pi}{2} - \cos \frac{3\pi}{2}$	−1
2π	$\sin 2\pi - \cos 2\pi$	−1



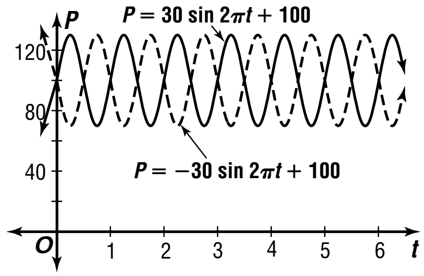
13a. $\frac{130 + 70}{2} = 100; P = 100$

13b. $|A| = \frac{130 - 70}{2} \qquad \frac{2\pi}{k} = 1$

$|A| = 30 \qquad k = 2\pi$

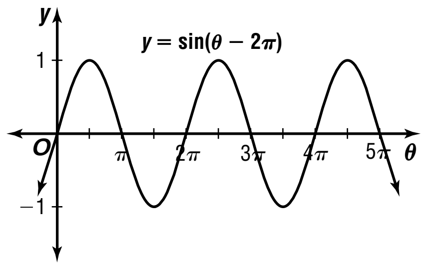
$A = \pm 30 \qquad P = \pm 30 \sin 2\pi t + 100$

13c.

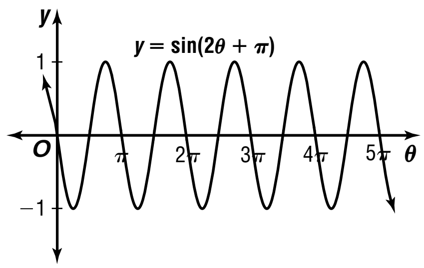


Pages 383–386 Exercises

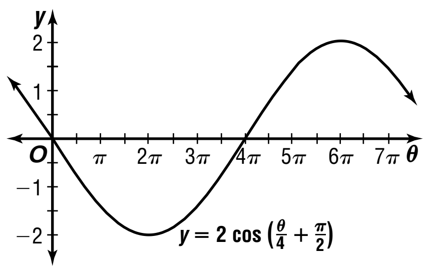
14. $-\frac{c}{k} = -\frac{-2\pi}{1}$ or $2\pi; A = 1; \frac{2\pi}{1} = 2\pi$



15. $-\frac{c}{k} = -\frac{\pi}{2}; A = 1; \frac{2\pi}{2} = \pi$

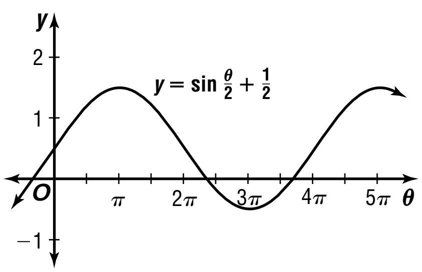


16. $-\frac{c}{k} = -\frac{\frac{\pi}{2}}{\frac{1}{4}}$ or $-2\pi; A = 2; \frac{2\pi}{\frac{1}{4}} = 8\pi$



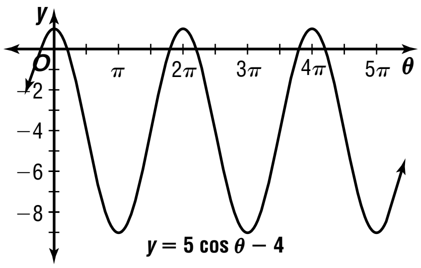
17. $\frac{1}{2}; y = \frac{1}{2}$

$A = 1; \frac{2\pi}{\frac{1}{2}} = 4\pi$



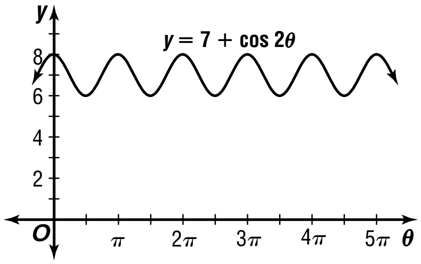
18. $-4; y = -4$

$A = 5; \frac{2\pi}{1} = 2\pi$



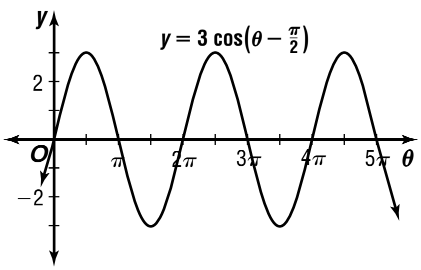
19. $7; y = 7$

$A = 1; \frac{2\pi}{2} = \pi$

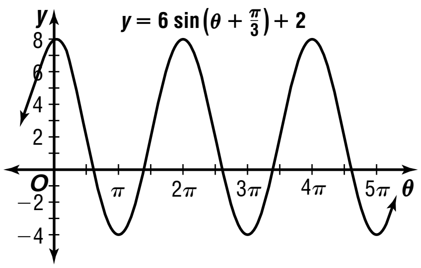


20. $-\frac{c}{k} = -\frac{-4\pi}{2}$ or $2\pi; -3$

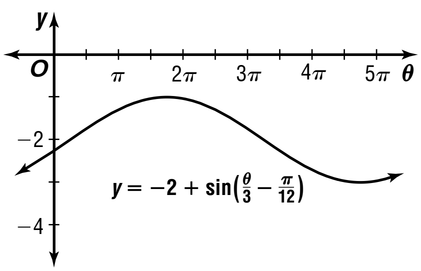
21. $|3| = 3; \frac{2\pi}{1} = 2\pi; \frac{-\frac{\pi}{2}}{1} = \frac{\pi}{2}; 0$



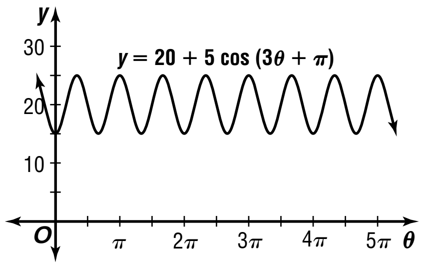
22. $|6| = 6; \frac{2\pi}{1} = 2\pi; -\frac{\frac{\pi}{3}}{1} = -\frac{\pi}{3}; 2$



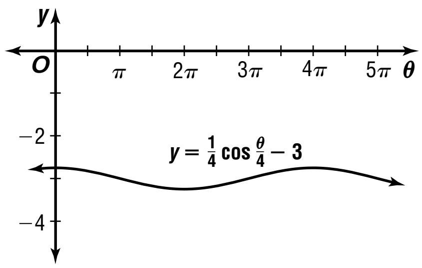
23. $|1| = 1; \frac{2\pi}{\frac{1}{3}} = 6\pi; -\frac{-\frac{\pi}{12}}{\frac{1}{3}} = \frac{\pi}{4}; -2$



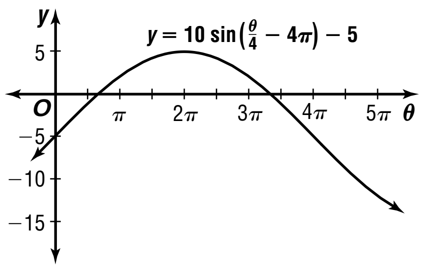
24. $|5| = 5; \frac{2\pi}{3}; -\frac{\pi}{3}; 20$



25. $\left|\frac{1}{4}\right| = \frac{1}{4}; \frac{2\pi}{\frac{1}{2}} = 4\pi; -\frac{0}{\frac{1}{2}} = 0; -3$



26. $|10| = 10; \frac{2\pi}{\frac{1}{4}} = 8\pi; -\frac{-4\pi}{\frac{1}{4}} = 16\pi; -5$



27. $|A| = \frac{2 - (-6)}{2}$ $\quad$ 4π; $\frac{\pi}{2}$ to the left or $-\frac{\pi}{2}$;

$|A| = 4$ $\quad$ down 2, or -2

$A = \pm 4; 4$

28. $|A| = 7 \quad \frac{2\pi}{k} = 3\pi \quad -\frac{c}{\frac{2}{3}} = \pi \quad h = -7$

$A = \pm 7 \quad k = \frac{2}{3} \quad c = -\frac{2\pi}{3}$

$y = \pm 7\sin\left(\frac{2}{3}\theta - \frac{2\pi}{3}\right) - 7$

29. $|A| = 50 \quad \frac{2\pi}{k} = \frac{3\pi}{4} \quad -\frac{c}{\frac{8}{3}} = \frac{\pi}{2} \quad h = -25$

$A = \pm 50 \quad k = \frac{8}{3} \quad c = -\frac{4\pi}{3}$

$y = \pm 50\sin\left(\frac{8}{3}\theta - \frac{4\pi}{3}\right) - 25$

30. $|A| = \frac{3}{4} \quad \frac{2\pi}{k} = \frac{\pi}{5} \quad -\frac{c}{10} = \pi \quad h = \frac{1}{4}$

$A = \pm\frac{3}{4} \quad k = 10 \quad c = -10\pi$

$y = \pm\frac{3}{4}\sin(10\theta - 10\pi) + \frac{1}{4}$

31. $|A| = 3.5 \quad \frac{2\pi}{k} = \frac{\pi}{2} \quad -\frac{c}{4} = \frac{\pi}{4} \quad h = 7$

$A = \pm 3.5 \quad k = 4 \quad c = -\pi$

$y = \pm 3.5\cos(4\theta - \pi) + 7$

32. $|A| = \frac{4}{5} \quad \frac{2\pi}{k} = \frac{\pi}{6} \quad -\frac{c}{12} = \frac{\pi}{3} \quad h = \frac{7}{5}$

$A = \pm\frac{4}{5} \quad k = 12 \quad c = -4\pi$

$y = \pm\frac{4}{5}\cos(12\theta - 4\pi) + \frac{7}{5}$

33. $|A| = 100 \quad \frac{2\pi}{k} = 45 \quad -\frac{c}{\frac{2\pi}{45}} = 0 \quad h = -110$

$A = \pm 100 \quad k = \frac{2\pi}{45} \quad c = 0$

$y = \pm 100 \cos\left(\frac{2\pi}{45}\theta\right) - 110$

34. $|A| = \frac{1-(-3)}{2} \quad \frac{2\pi}{k} = 4\pi \quad h = -1$

$|A| = 2 \quad k = \frac{1}{2}$

$A = \pm 2; -2$

$y = -2 \cos\left(\frac{\theta}{2}\right) - 1$

35. $|A| = \frac{3.5-(2.5)}{2} \quad \frac{2\pi}{k} = \pi \quad -\frac{c}{2} = 0 \quad h = 3$

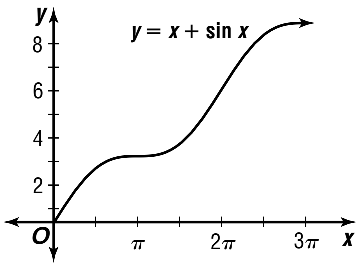
$|A| = 0.5 \quad k = 2 \quad c = 0$

$A = \pm 0.5; 0.5$

$y = 0.5 \sin 2\theta + 3$

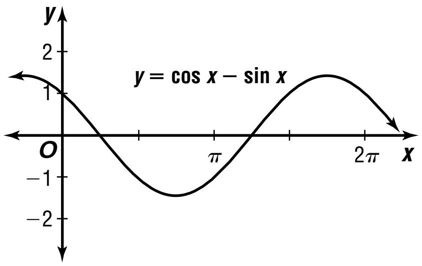
36.

x	$\sin x$	$\sin x + x$
0	0	0
$\frac{\pi}{2}$	1	$\frac{\pi}{2} + 1 \approx 2.57$
π	0	$\pi \approx 3.14$
$\frac{3\pi}{2}$	-1	$\frac{3\pi}{2} - 1 \approx 3.71$
2π	0	$2\pi \approx 6.28$



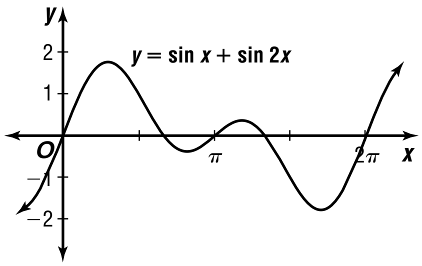
37.

x	$\cos x$	$\sin x$	$\cos x - \sin x$
0	1	0	1
$\frac{\pi}{2}$	0	1	-1
π	-1	0	-1
$\frac{3\pi}{2}$	0	-1	1
2π	1	0	1

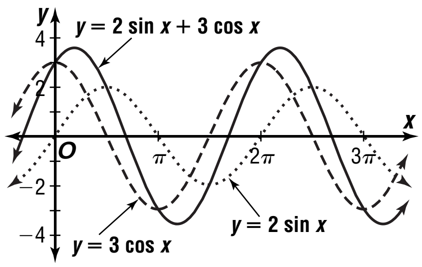


38.

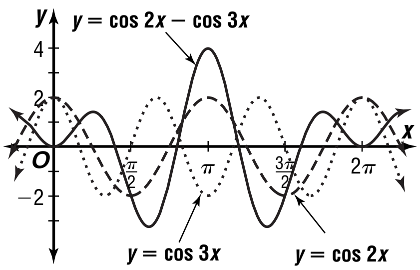
x	$\sin x$	$2x$	$\sin 2x$	$\sin x + \sin 2x$
0	0	0	0	0
$\frac{\pi}{4}$	0.71	$\frac{\pi}{2}$	1	1.71
$\frac{\pi}{2}$	1	π	0	1
π	0	2π	0	0
$\frac{3\pi}{2}$	-1	3π	0	-1
2π	0	4π	0	0



39.



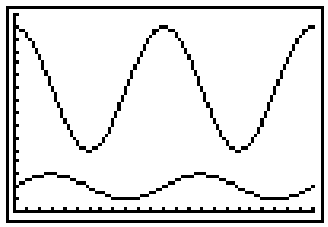
40.



41a. $2000 + 1000 = 3000$
$2000 - 1000 = 1000$

41b. $10{,}000 + 5000 = 15{,}000$
$10{,}000 - 5000 = 5000$

41c.



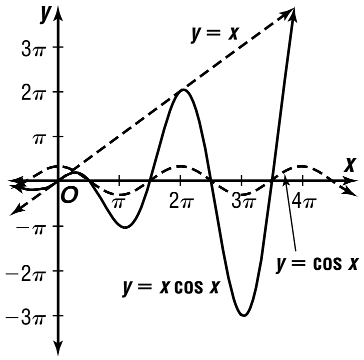
[0, 24] sc1:1 by [0, 16,000] sc1:1000

41d. months number 3 and 15

41e. months number 0, 12, 24

41f. When the sheep population is at a maximum, the wolf population is on the increase because of the maximum availability of food. The upswing in wolf population leads to a maximum later.

42.



43a. $46 - 42 = 4$ ft

43b. $r = \frac{1}{2}d$ $\qquad t = 21 + 4$

$r = \frac{1}{2}(42)$ $\qquad t = 25$

$r = 21$

43c. $\frac{3 \text{ revolutions}}{60 \text{ seconds}} = \frac{1 \text{ revolution}}{x \text{ seconds}}$

$x = 20$ s

43d. $|A| = 21$ $\qquad \frac{2\pi}{k} = 20$ $\qquad h = 25$

$A = \pm 21; 21$ $\qquad k = \frac{\pi}{10}$

$h = 25 + 21 \sin \frac{\pi t}{10}$

43e. $h = 25 + 21 \sin\left(\frac{\pi t}{10}\right)$

$46 = 25 + 21 \sin\left(\frac{\pi t}{10}\right)$

$1 = \sin\left(\frac{\pi t}{10}\right)$

$\sin^{-1} = \sin\left(\frac{\pi t}{10}\right)$

$5 = t$; 5 s

43f. $h = 25 + 21 \sin\left(\frac{\pi t}{10}\right)$

$h = 25 + 21 \sin\left(\frac{\pi \cdot 10}{10}\right)$

$h = 25$ ft

44. $-\frac{c}{k} = -\frac{0}{2}$ or 0

$-\frac{c}{k} = -\frac{-\frac{\pi}{2}}{2}$ or $\frac{\pi}{4}$

There is a $\frac{\pi}{4}$ phase difference.

45a. $y = \sqrt{\sin x}$ $\qquad$ **45b.** $y = \frac{\cos x}{x}$

45c. $y = \cos x^2$ $\qquad$ **45d.** $y = \sin \sqrt{x}$

46. $\frac{2\pi}{k} = \frac{1}{294}$

$k = 588\pi$ $\qquad y = 0.25 \sin 588\pi t$

47. $v = r\omega$

$v = 7(19.2)$

$v = 134.4$ cm/s

48. asymptote: $x = 2$

$y = \frac{x-3}{x-2}$

$y(x-2) = x - 3$

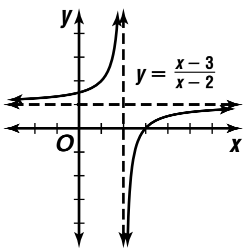
$yx - 2y = x - 3$

$-2y - 3 = x - yx$

$-2y - 3 = x(1 - y)$

$\frac{-2y-3}{1-y} = x$

asymptote: $y = 1$



49. $f(x) = \frac{3}{x-1}$

$y = \frac{3}{x-1}$

$x = \frac{3}{y-1}$

$x(y-1) = 3$

$y - 1 = \frac{3}{x}$

$y = \frac{3}{x} + 1$

50. $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ -3 & -5 \end{bmatrix} = X$

$\begin{bmatrix} 1(3) + 1(-3) & 1(5) + 1(-5) \\ 1(3) + 1(-3) & 1(5) + 1(-5) \end{bmatrix} = X$

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = X$

51. $7(3x + 5y) = 7(4)$ $\rightarrow$ $21x + 35y = 28$

$14x - 35y = 21$ $\qquad 14x - 35y = 21$

$35x = 49$

$x = 1.4$

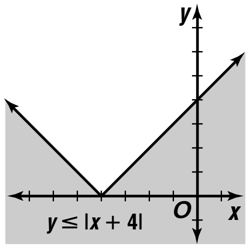
$3x + 5y = 4$

$3(1.4) + 5y = 4$

$y = -0.04$ $\qquad (1.4, -0.04)$

52.

x	$\|x + 4\|$	y
−6	$\|-6 + 4\|$	2
−4	$\|-4 + 4\|$	0
−2	$\|-2 + 4\|$	2
0	$\|0 + 4\|$	4



53. $3x - y + 7 = 0$ $\qquad y - y_1 = m(x - x_1)$

$y = 3x + 7$ $\qquad y - (-2) = 3(x - 3)$

slope: 3 $\qquad y + 2 = 3x - 9$

$3x - y - 11 = 0$

54. 4 inches $= \frac{1}{3}$ foot

$75 \times 42 \times \frac{1}{3} = 1050$ cubic feet

$1050 \times 7.48 = 7854$ gal

The correct answer is 7854.

6-6 Modeling Real-World Data with Sinusoidal Functions

Pages 390–391 Check for Understanding

1. any function that can be written as a sine function or a cosine function
2. Both data that can be modeled with a polynomial function and data that can be modeled with a sinusoidal function have fluctuations. However, data that can be modeled with a sinusoidal function repeat themselves periodically, and data that can be modeled with a polynomial function do not.
3. Sample answers: the amount of daylight, the average monthly temperatures, the height of a seat on a Ferris wheel

4a. $y = -5\cos\left(\frac{\pi}{6}t\right)$

$y = -5\cos\left(\frac{\pi}{6}\cdot 0\right)$

$y = -5$

5 units below equilibrium

4b. 5 units above equilibrium

4c. $y = -5\cos\left(\frac{\pi}{6}t\right)$

$y = -5\cos\left(\frac{\pi}{6}\cdot 7\right)$

$y = -4.33$

about 4.33 units above equilibrium

5. $A = \frac{140 - 80}{2}$ $\quad h = \frac{140 + 80}{2}$

$A = 30$ $\quad h = 110$

$\frac{2\pi}{k} = 1$

$k = 2\pi$ $\quad P = 30\sin 2\pi t + 110$

6a. $A = \frac{66° - 41°}{2}$

$A = 12.5°$

6b. $h = \frac{66° + 41°}{2}$

$h = 53.5°$

6c. 12 months

6d. $A = \pm 12.5$ $\quad \frac{2\pi}{k} = 12$ $\quad h = 53.5$

$k = \frac{\pi}{6}$

$y = -12.5\cos\left(\frac{\pi}{6}t + c\right) + 53.5$

$41 = -12.5\cos\left(\frac{\pi}{6}\cdot 1 + c\right) + 53.5$

$-12.5 = -12.5\cos\left(\frac{\pi}{6} + c\right)$

$1 = \cos\left(\frac{\pi}{6} + c\right)$

$\cos^{-1} 1 = \frac{\pi}{6} + c$

$\cos^{-1} 1 - \frac{\pi}{6} = c$

$-0.5 \approx c$

Sample answer: $y = -12.5\cos\left(\frac{\pi}{6}t - 0.5\right) + 53.5$

6e. $y = -12.5\cos\left(\frac{\pi}{6}t - 0.5\right) + 53.5$

$y = -12.5\cos\left(\frac{\pi}{6}(2) - 0.5\right) + 53.5$

$y \approx 42.82517529$

Sample answer: About 42.8°; it is somewhat close to the actual average.

6f. $y = -12.5\cos\left(\frac{\pi}{6}t - 0.5\right) + 53.5$

$y = -12.5\cos\left(\frac{\pi}{6}(10) - 0.5\right) + 53.5$

$y \approx 53.20504268$

Sample answer: About 53.2°; it is close to the actual average.

Pages 391–394 Exercises

7a. 0.5

7b. $\frac{2\pi}{k} = 660\pi$

$k = \frac{1}{330}$

7c. $\frac{1}{\frac{1}{330}} = 330$ hertz

8a. $3.5 + |-3| = 6.5$ units

8b. $3.5 - 3 = 0.5$ units

8c. $\frac{2\pi}{k} = \frac{2\pi}{\frac{5\pi}{3}}$ or $\frac{6}{5}$

8d. $h = -3\cos\left(\frac{5\pi}{3}t\right) + 3.5$

$h = -3\cos\left(\frac{5\pi}{3}(25)\right) + 3.5$

$h = 2$ units

9a. $R = 1200 + 300\sin\left(\frac{\pi}{2}t\right)$

$R = 1200 + 300\sin\left(\frac{\pi}{2}\cdot 0\right)$

$R = 1200$

9b. $H = 250 + 25\sin\left(\frac{\pi}{2}t - \frac{\pi}{4}\right)$

$H = 250 + 25\sin\left(\frac{\pi}{2}\cdot 0 - \frac{\pi}{4}\right)$

$H \approx 232$

9c. R: $1200 + 300 = 1500$

H: $250 + 25 = 275$ no

9d. $R = 1200 + 300\sin\left(\frac{\pi}{2}t\right)$

$1500 = 1200 + 300\sin\left(\frac{\pi}{2}t\right)$

$300 = 300\sin\frac{\pi}{2}t$

$1 = \sin\frac{\pi}{2}t$

$\sin^{-1} 1 = \frac{\pi}{2}t$

$\sin^{-1}1\left(\frac{2}{\pi}\right) = t$

$1 = t$

January 1, 1971

9e. $250 - 25 = 225$

$H = 250 + 25\sin\left(\frac{\pi}{2}t - \frac{\pi}{4}\right)$

$225 = 250 + 25\sin\left(\frac{\pi}{2}t - \frac{\pi}{4}\right)$

$-25 = 25\sin\left(\frac{\pi}{2}t - \frac{\pi}{4}\right)$

$-1 = \sin\left(\frac{\pi}{2}t - \frac{\pi}{4}\right)$

$\sin^{-1} - 1 = \frac{\pi}{2}t - \frac{\pi}{4}$

$\frac{2}{\pi}\left(\sin^{-1} - 1 + \frac{\pi}{4}\right) = t$

$-0.5 = t$

July 1, 1969; $\frac{2\pi}{k} = \frac{2\pi}{\frac{\pi}{2}}$

$k = 4$

next minimum: July 1, 1973

9f. See students' work.

10. $A = \frac{4}{2}$ $\quad \frac{2\pi}{k} = 10$

$A = 2$ $\quad k = \frac{\pi}{5}$

$y = 2\cos\left(\frac{\pi}{5}t\right)$

11. $h = 4.25$; $A = 3.55$; $\frac{2\pi}{k} = 12.40$; $-\frac{c}{\frac{\pi}{6.2}} = -4.68$

$k = \frac{\pi}{6.2}$ $\quad c = \frac{2.34\pi}{3.1}$

$y = 3.55\sin\left(\frac{\pi}{6.2}t + \frac{2.34\pi}{3.1}\right) + 4.24$

12a. $h = 47.5$; $A = 23.5$; $\frac{2\pi}{k} = 12$; $-\frac{c}{\frac{\pi}{6}} = 4$

$k = \frac{\pi}{6}$ $\quad c = -\frac{2\pi}{3}$

$y = 23.5\sin\left(\frac{\pi}{6}t - \frac{2\pi}{3}\right) + 47.5$

12b. $y = 23.5 \sin\left(\frac{\pi}{6}t - \frac{2\pi}{3}\right) + 47.5$

$y = 23.5 \sin\left(\frac{\pi}{6} \cdot 3 - \frac{2\pi}{3}\right) + 47.5$

$y = 35.75$

about 35.8°

12c. $y = 23.5 \sin\left(\frac{\pi}{6}t - \frac{2\pi}{3}\right) + 47.5$

$y = 23.5 \sin\left(\frac{\pi}{6} \cdot 8 - \frac{2\pi}{3}\right) + 47.5$

$y \approx 67.9°$

13a. $A = \frac{81° - 73°}{2}$

$A = 4°$

13b. $h = \frac{81° + 73°}{2}$

$h = 77°$

13c. 12 months

13d. $A = \pm 4$ $\quad \frac{2\pi}{k} = 12 \quad h = 77$

$k = \frac{\pi}{6}$

$y = -4 \cos\left(\frac{\pi}{6}t + c\right) + 77$

$73 = -4 \cos\left(\frac{\pi}{6} \cdot 1 + c\right) + 77$

$-4 = -4 \cos\left(\frac{\pi}{6} + c\right)$

$1 = \cos\left(\frac{\pi}{6} + c\right)$

$\cos^{-1} 1 = \frac{\pi}{6} + c$

$\cos^{-1} 1 - \frac{\pi}{6} = c$

$-0.5235987756 \approx c$

Sample answer: $y = -4 \cos\left(\frac{\pi}{6}t - 0.5\right) + 77$

13e. $y = -4 \cos\left(\frac{\pi}{6}t - 0.5\right) + 77$

$y = -4 \cos\left(\frac{\pi}{6} \cdot 8 - 0.5\right) + 77$

$y \approx 80.41594391$

Sample answer: About 80.4°; it is very close to the actual average.

13f. $y = -4 \cos\left(\frac{\pi}{6}t - 0.5\right) + 77$

$y = -4 \cos\left(\frac{\pi}{6} \cdot 5 - 0.5\right) + 77$

$y \approx 79.08118409$

Sample answer: About 79.1°; it is close to the actual average.

14. $-\frac{c}{k} = -\frac{-\pi}{1}$ or π

increase shift by $\frac{\pi}{2}$; $\pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$-\frac{c}{k} = \frac{3\pi}{2}$

$-\frac{c}{1} = \frac{3\pi}{2}$

$c = -\frac{3\pi}{2}$

Sample answer: $y = 3 \cos\left(x - \frac{3\pi}{2}\right) + 5$

15a. $A = \frac{13.25 - 1.88}{2}$

$A = 5.685$ ft

15b. $h = \frac{13.25 + 1.88}{2}$

$h = 7.565$ ft

15c. 4:53 P.M. − 4:30 A.M. = 12:23 or about 12.4 h

15d. $A = \pm 5.685$ $\quad \frac{2\pi}{k} = 12.4 \quad h = 7.565$

$k = \frac{\pi}{6.2}$

4:30 A.M. = 4.5 hrs

$h = 5.685 \sin\left(\frac{\pi}{6.2}t + c\right) + 7.565$

$13.25 = 5.685 \sin\left(\frac{\pi}{6.2} \cdot 4.5 + c\right)$

$5.685 = 5.685 \sin\left(\frac{4.5\pi}{6.2} + c\right)$

$1 = \sin\left(\frac{4.5\pi}{6.2} + c\right)$

$\sin^{-1} 1 = \frac{4.5\pi}{6.2} + c$

$\sin^{-1} 1 - \frac{4.5\pi}{6.2} = c$

$-0.7093918895 \approx c$

Sample answer: $h = 5.685 \sin\left(\frac{\pi}{6.2}t - 0.71\right) + 7.565$

15e. 7:30 P.M. = 19.5 hrs

$h = 5.685 \sin\left(\frac{\pi}{6.2}t - 0.71\right) + 7.565$

$h = 5.685 \sin\left(\frac{\pi}{6.2} \cdot 19.5 - 0.71\right) + 7.565$

$h \approx 8.993306129$

Sample answer: about 8.99 ft

16a. Table at bottom of page.

Month	Sunrise A.M.	A.M. Time in Decimals	Sunset P.M.	P.M. Time in Decimals	Daylight Hours (P.M.-A.M.)
January	7:19	7.317	4:47	16.783	9.47 h
February	6:56	6.933	5:24	17.4	10.47 h
March	6:16	6.267	5:57	17.95	11.68 h
April	5:25	5.416	6:29	18.483	13.07 h
May	4:44	4.733	7:01	19.017	14.28 h
June	4:24	4.4	7:26	19.433	15.03 h
July	4:33	4.55	7:28	19.467	14.92 h
August	5:01	5.017	7:01	19.017	14 h
September	5:31	5.517	6:14	18.233	12.72 h
October	6:01	6.017	5:24	17.4	11.38 h
November	6:36	6.6	4:43	16.717	10.12 h
December	7:08	7.133	4:28	16.467	9.33 h

16b. $A = \frac{15.03 - 9.33}{2}$

$A = 2.85$ h

16c. $h = \frac{15.03 + 9.33}{2}$

$h = 12.18$ h

16d. 12 months

16e. $A = \pm 2.85$ $\quad \frac{2\pi}{k} = 12 \quad h = 12.18$

$k = \frac{\pi}{6}$

$$y = -2.85 \cos\left(\frac{\pi}{6}t + c\right) + 12.18$$

$$9.47 = -2.85 \cos\left(\frac{\pi}{6} \cdot 1 + c\right) + 12.18$$

$$-2.71 = -2.85 \cos\left(\frac{\pi}{6} + c\right)$$

$$0.950877193 \approx \cos\left(\frac{\pi}{6} + c\right)$$

$$\cos^{-1} 0.950877193 \approx \frac{\pi}{6} + c$$

$$\cos^{-1} 0.950877193 - \frac{\pi}{6} \approx c$$

$$-0.2088597251 \approx c$$

Sample answer: $y = -2.85 \cos\left(\frac{\pi}{6}t - 0.21\right) + 12.18$

17. $70.5 - 19.5 = 51$

$$y = 70.5 + 19.5 \sin\left(\frac{\pi}{6}t + c\right)$$

$$51 = 70.5 + 19.5 \sin\left(\frac{\pi}{6} \cdot 1 + c\right)$$

$$-19.5 = 19.5 \sin\left(\frac{\pi}{6} + c\right)$$

$$-1 = \sin\left(\frac{\pi}{6} + c\right)$$

$$\sin^{-1} - 1 = \frac{\pi}{6} + c$$

$$\sin^{-1} - 1 - \frac{\pi}{6} = c$$

$$-2.094395102 \approx c$$

Sample answer: about -2.09

18a. $\frac{14 \text{ revolutions}}{1 \text{ minute}} \times \frac{1 \text{ minute}}{60 \text{ seconds}} \times \frac{2\pi \text{ radians}}{1 \text{ revolution}} = \frac{7\pi}{15}$ rad/s

$y = -3.5 \cos\left(\frac{7\pi}{15}t\right)$

18b. $y = -3.5 \cos\left(\frac{7\pi}{15}t\right)$

$y = -3.5 \cos\left(\frac{7\pi}{15} \cdot 4\right)$

$y \approx -3.197409102$

about $(4, -3.20)$

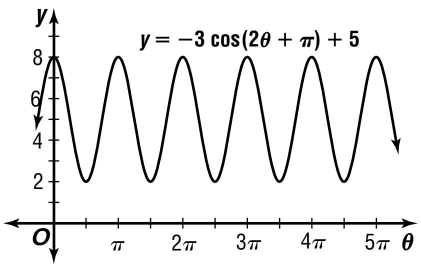
19. $A = \frac{120 - (-120)}{2}$ $\quad \frac{2\pi}{k} = 60$

$A = 120$ $\quad k = \frac{\pi}{30}$

$V_R = 120 \sin\left(\frac{\pi}{30}t\right)$

20. See students' work.

21. $|-3| = 3$; $\frac{2\pi}{2} = \pi$; $-\frac{\pi}{2}$; 5



22. $2\pi n$ where n is an integer

23. $800° \times \frac{\pi}{180°} = \frac{40\pi}{9}$

24. $40^2 = 32^2 + 20^2 - 2(32)(20) \cos \theta$

$\cos \theta = \frac{40^2 - 32^2 - 20^2}{-2(32)(20)}$

$\theta = \cos^{-1}\left(\frac{40^2 - 32^2 - 20^2}{-2(32)(20)}\right)$

$\theta \approx 97.9°$

$180° - 97.9° = 82.1°$

about 97.9°, 82.1°, 97.9°, 82.1°

25. $\frac{2m + 16}{m^2 - 16} = \frac{2m + 16}{(m - 4)(m + 4)}$

$\frac{2m + 16}{(m - 4)(m + 4)} = \frac{A}{m - 4} + \frac{B}{m + 4}$

$2m + 16 = A(m + 4) + B(m - 4)$

Let $m = -4$.

$2(-4) + 16 = A(-4 + 4) + B(-4 - 4)$

$8 = -8B$

$-1 = B$

Let $m = 4$.

$2(4) + 16 = A(4 + 4) + B(4 - 4)$

$24 = 8A$

$3 = A$

$\frac{A}{m - 4} + \frac{B}{m + 4} = \frac{3}{m - 4} + \frac{-1}{m + 4}$

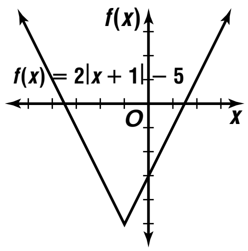
26.

-2	2	k	-1	-6
		-4	$-2k + 8$	$4k - 14$
	2	$k - 4$	$-2k + 7$	$4k - 20$

$4k - 20 = 0$

$k = 5$

27.



increasing: $x > -1$; decreasing: $x < -1$

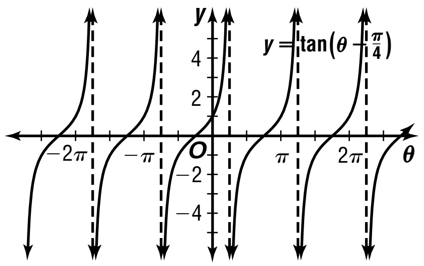
28. The correct choice is E.

6-7 Graphing Other Trigonometric Functions

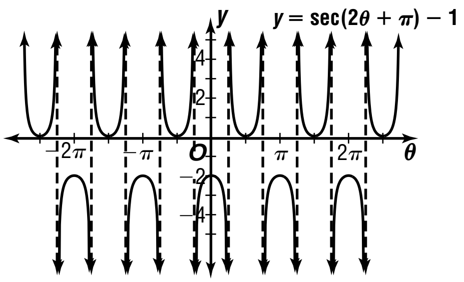
Page 400 Check for Understanding

1. Sample answers: $-\pi$, π, 2π
2. The asymptotes of $y = \tan \theta$ and $y = \sec \theta$ are the same. The period of $y = \tan \theta$ is π and the period of $y = \sec \theta$ is 2π.
3. Sample answers: $\frac{\pi}{2}$, $-\frac{3\pi}{2}$
4. 0
5. 1
6. πn, where n is an odd integer
7. $\frac{\pi}{4} + \pi n$, where n is an integer

8. $\frac{\pi}{1} = \pi$; $\frac{-\frac{\pi}{4}}{1} = -\frac{\pi}{4}$



9. $\frac{2\pi}{2} = \pi$; $-\frac{\pi}{2}$; $h = -1$



10. k: $\frac{2\pi}{k} = 3\pi$ $\quad$ c: $-\frac{c}{\frac{2}{3}} = \frac{\pi}{3}$ $\quad$ h: $h = -4$

$k = \frac{2}{3}$

$c = -\frac{2\pi}{9}$

$y = \csc\left(\frac{2}{3}\theta - \frac{2\pi}{9}\right) - 4$

11. k: $\frac{\pi}{k} = 2\pi$ $\quad$ c: $-\frac{c}{\frac{1}{2}} = -\frac{\pi}{4}$ $\quad$ h: $h = 0$

$k = \frac{1}{2}$

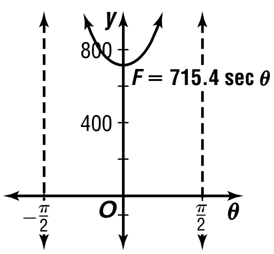
$c = \frac{\pi}{8}$

$y = \cot\left(\frac{1}{2}\theta + \frac{\pi}{8}\right)$

12a. $f = ma$
$f = 73(9.8)$
$f = 715.4$ N

12b. $F = f \sec \theta$
$F = 715.4 \sec \theta$

12c. $\frac{2\pi}{1} = 2\pi$; no phase shift, no vertical shift



12d. 715.4 N

12e. The tension becomes greater.

Pages 400–403 Exercises

13. 0

14. 0

15. undefined

16. -1

17. -1

18. undefined

19. undefined

20. 0

21. πn, where n is an integer

22. πn, where n is an even integer

23. $\frac{3\pi}{2} + 2\pi n$, where n is an integer

24. $\frac{\pi}{4} + \pi n$, where n is an integer

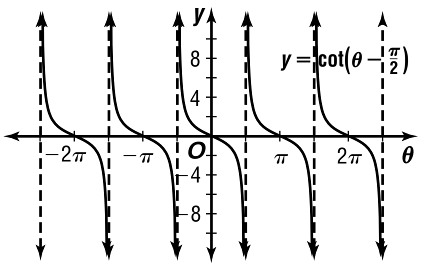
25. $-\frac{\pi}{4} + \pi n$, where n is an integer

26. $\frac{3\pi}{4} + \pi n$, where n is an integer

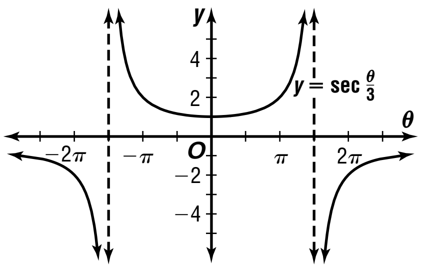
27. $\frac{\pi}{2}n$, where n is an odd integer

28. πn, where n is an integer

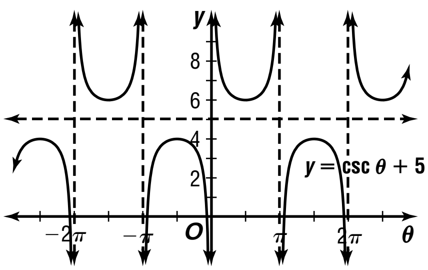
29. $\frac{\pi}{1} = \pi$; $-\frac{-\frac{\pi}{2}}{1} = \frac{\pi}{2}$



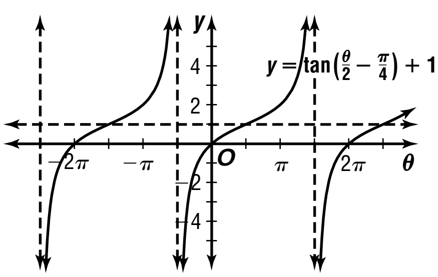
30. $\frac{2\pi}{\frac{1}{3}} = 6\pi$; no phase shift; no vertical shift



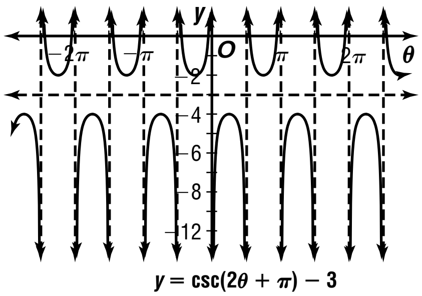
31. $\frac{2\pi}{1} = 2\pi$; no phase shift; $h = 5$



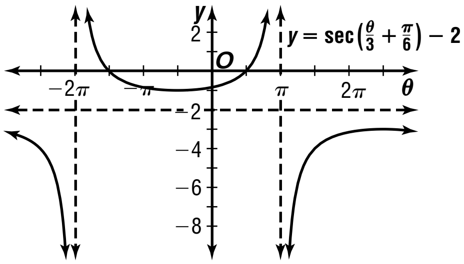
32. $\frac{\pi}{\frac{1}{2}} = 2\pi$; $-\frac{-\frac{\pi}{4}}{\frac{1}{2}} = \frac{\pi}{2}$; $h = 1$



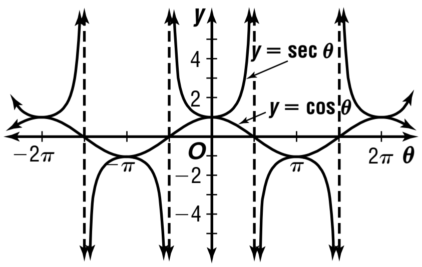
33. $\frac{2\pi}{2} = \pi$; $-\frac{\pi}{2}$; $h = -3$



34. $\frac{2\pi}{\frac{1}{3}} = 6\pi$; $-\frac{\frac{\pi}{6}}{\frac{1}{3}} = -\frac{\pi}{2}$; $h = -2$



35.



$-2\pi, -\pi, 0, \pi, 2\pi$

36. k: $\frac{\pi}{k} = 2\pi$ c: $-\frac{c}{\frac{1}{2}} = 0$ h: $h = -6$

$k = \frac{1}{2}$ $c = 0$

$y = \tan\frac{\theta}{2} - 6$

37. k: $\frac{\pi}{k} = \frac{\pi}{2}$ c: $-\frac{c}{2} = \frac{\pi}{8}$ h: $h = 7$

$k = 2$ $c = -\frac{\pi}{4}$

$y = \cot\left(2\theta - \frac{\pi}{4}\right) + 7$

38. k: $\frac{2\pi}{k} = \pi$ c: $-\frac{c}{2} = -\frac{\pi}{4}$ h: $h = -10$

$k = 2$ $c = \frac{\pi}{2}$

$y = \sec\left(2\theta + \frac{\pi}{2}\right) - 10$

39. k: $\frac{2\pi}{k} = 3\pi$ c: $-\frac{c}{\frac{2}{3}} = \pi$ h: $h = -1$

$k = \frac{2}{3}$ $c = -\frac{2\pi}{3}$

$y = \csc\left(\frac{2}{3}\theta - \frac{2\pi}{3}\right) - 1$

40. k: $\frac{\pi}{k} = 5\pi$ c: $-\frac{c}{\frac{1}{5}} = -\pi$ h: $h = 12$

$k = \frac{1}{5}$ $c = \frac{\pi}{5}$

$y = \cot\left(\frac{\theta}{5} + \frac{\pi}{5}\right) + 12$

41. k: $\frac{2\pi}{k} = \frac{\pi}{3}$ c: $-\frac{c}{6} = -\frac{\pi}{2}$ h: $h = -5$

$k = 6$ $c = 3\pi$

$y = \csc(6\theta + 3\pi) - 5$

42. k: $\frac{2\pi}{k} = 3\pi$ c: $-\frac{c}{\frac{2}{3}} = -\pi$ h: $h = -8$

$k = \frac{2}{3}$ $c = \frac{2\pi}{3}$

$y = \sec\left(\frac{2}{3}\theta + \frac{2\pi}{3}\right) - 8$

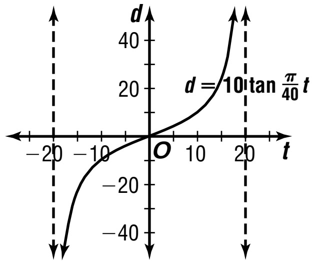
43. k: $\frac{\pi}{k} = \frac{\pi}{2}$ c: $-\frac{c}{2} = \frac{\pi}{4}$ h: $h = 7$

$k = 2$ $c = -\frac{\pi}{2}$

$y = \tan\left(2\theta - \frac{\pi}{2}\right) + 7$

44a. $\frac{\pi}{\frac{\pi}{40}} = 40$

no phase shift
no vertical shift



44b. $d = 10 \tan \frac{\pi}{40} t$

$d = 10 \tan \frac{\pi}{40}(3)$

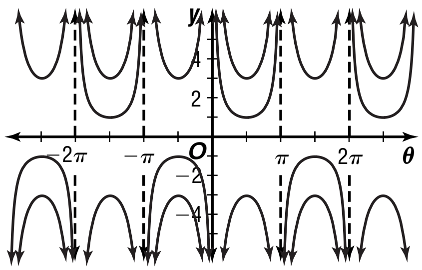
$d \approx 2.4$ ft from the center

44c. $d = 10 \tan \frac{\pi}{40} t$

$d = 10 \tan \frac{\pi}{40}(15)$

$d \approx 24.1$ ft from the center

45.

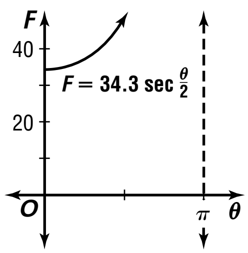


The graph of $y = \csc\theta$ has no range values between -1 and 1, while the graphs of $y = 3\csc\theta$ and $y = -3\csc\theta$ have no range values between -3 and 3. The graphs of $y = 3\csc\theta$ and $y = -3\csc\theta$ are reflections of each other.

46a. $f = m \cdot 9.8$
$f = 7 \cdot 9.8$
$f = 68.6$ N

46b. $F = \frac{1}{2} f \sec\frac{\theta}{2}$
$F = \frac{1}{2}(68.6) \sec\frac{\theta}{2}$
$F = 34.3 \sec\frac{\theta}{2}$

46c. $\frac{2\pi}{\frac{1}{2}} = 4\pi$



46d. 34.3 N

46e. The tension becomes greater.

47a. 220 A

47b. $\frac{2\pi}{60\pi} = \frac{1}{30}$ s

47c. $-\frac{-\frac{\pi}{6}}{60\pi} = \frac{1}{360}$

47d. $I = 220 \sin\left(60\pi t - \frac{\pi}{6}\right)$

$I = 220 \sin\left(60\pi \cdot 60 - \frac{\pi}{6}\right)$

$I \approx -110$ A

48. $y = -1 \tan\left(\theta + \frac{\pi}{2}\right)$

49a. $A = \frac{3.99 - 0.55}{2}$

$A = 1.72$ ft

49b. $h = \frac{3.99 + 0.55}{2}$

$h = 2.27$ ft

49c. 12:19 P.M. − 12:03 A.M. = 12:16 or about 12.3 hr

49d. $A = \pm 1.72$ $\quad \frac{2\pi}{k} = 12.3$ $\quad h = 2.27$

$k = \frac{2\pi}{12.3}$

12:03 = 0.05 hr since midnight

$h = 1.72 \sin\left(\frac{2\pi}{12.3} t + c\right) + 2.27$

$3.99 = 1.72 \sin\left(\frac{2\pi}{12.3} \cdot 0.05 + c\right) + 2.27$

$1.72 = 1.72 \sin\left(\frac{0.1\pi}{12.3} + c\right)$

$1 = \sin\left(\frac{0.1\pi}{12.3} + c\right)$

$\sin^{-1} 1 = \frac{0.1\pi}{12.3} + c$

$\sin^{-1} 1 - \frac{0.1\pi}{12.3} = c$

$1.545254923 \approx c$

Sample answer: $h = 1.72 \sin\left(\frac{2\pi}{12.3} t + 1.55\right) + 2.27$

49e. noon = 12 hrs since midnight

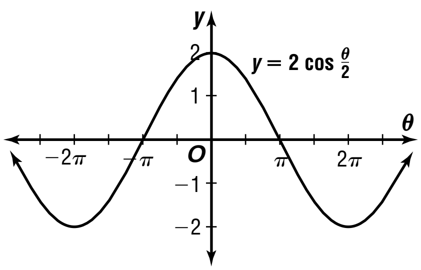
$h = 1.72 \sin\left(\frac{2\pi}{12.3} t + 1.55\right) + 2.27$

$h = 1.72 \sin\left(\frac{2\pi}{12.3} \cdot 12 + 1.55\right) + 2.27$

$h \approx 3.964014939$

Sample answer: 3.96 ft

50. $|2| = 2$; $\frac{2\pi}{\frac{1}{2}} = 4\pi$



51. $s = r\theta$

$s = 18\left(\frac{\pi}{3}\right)$

$s = 6\pi$ cm

52. $C = 180° - (62°31' + 75°18')$ or $42°11'$

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{57.3}{\sin 62°31'} = \frac{b}{\sin 75°18'}$

$b = \frac{57.3 \sin 75°18'}{\sin 62°31'}$

$b \approx 62.47505783$

$\frac{a}{\sin A} = \frac{c}{\sin C}$

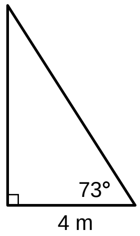
$\frac{57.3}{\sin 62°31'} = \frac{c}{\sin 42°11'}$

$c = \frac{57.3 \sin 42°11'}{\sin 62°31'}$

$c \approx 43.37198044$

$C = 42°11'$, $b = 62.5$, $c = 43.4$

53a.



53b. $\tan 73° = \frac{x}{4}$

$x = 4 \tan 73°$

$x \approx 13.1$ m

53c. $\cos 73° = \frac{4}{y}$

$y = \frac{4}{\cos 73°}$

$y \approx 13.7$ m

54. $a^2 + b^2 = c^2$

$7^2 + 4^2 = c^2$

$\sqrt{65} = c$

$\cos A = \frac{b}{c}$

$\cos A = \frac{4}{\sqrt{65}}$

$\cos A = \frac{4\sqrt{65}}{65}$

$\sin A = \frac{a}{c}$

$\sin A = \frac{7}{\sqrt{65}}$

$\sin A = \frac{7\sqrt{65}}{65}$

$\tan A = \frac{a}{b}$

$\tan A = \frac{7}{4}$

55. $\frac{x^2 - 4}{x^2 - 3x - 10} \le 0$

$\frac{(x-2)(x+2)}{(x-5)(x+2)} \le 0$

zeros: 2, −2

excluded values: 5, −2

Test −3: $\frac{(-3)^2 - 4}{(-3)^2 - 3(-3) - 10} \le 0$

$\frac{5}{5} \le 0$ false

Test 0: $\frac{0^2 - 4}{0^2 - 3(0) - 10} \le 0$

$\frac{-4}{-10} \le 0$ false

Test 3: $\frac{3^2 - 4}{3^2 - 3(3) - 10} \le 0$

$\frac{5}{-10} \le 0$ true

Test 6: $\frac{6^2 - 4}{6^2 - 3(6) - 10} \le 0$

$\frac{32}{8} \le 0$ false

$-2 < x < 5$

56. $k = \frac{6}{0.5}$

$k = 12$

$t = kr$

$10 = 12r$

$r \approx 0.83$

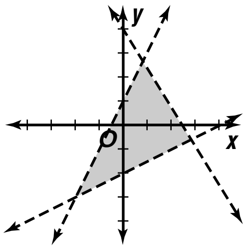
57. $3x + 2y < 8$

$y < -\frac{3}{2}x + 4$

$y < 2x + 1$

$-2y < -x + 4$

$y > \frac{1}{2}x - 2$



58. $y = 17.98x + 35.47$; 0.88

59. A: impossible to tell

B: $6(150) \overset{?}{\geq} 10(90)$

$900 = 900$; true

C: impossible to tell

D: $150 \overset{?}{\geq} 30 + 2(90)$

$150 = 210$; false

E: $3(90) \overset{?}{\geq} 30 + 2(150)$

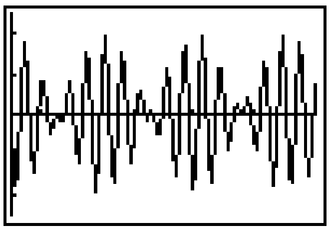
$270 = 330$; false

The correct choice is B.

6-7B Sound Beats

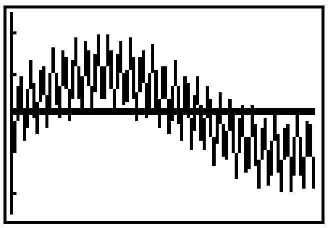
Page 404

1.



the third graph

2.



Sample answer: The graph seems to stay above the x-axis for an interval of x values, and then stay below the x-axis for another interval of x values.

3. 0.38623583

4. no

5. -1.78043; yes; the value for $f(x)$ is negative and corresponds to a point not graphed by the calculator.

6. Sample answer: As you move 1 pixel to the left or right of any pixel on the screen, the x-value for the adjacent pixel decreases or increases by almost 7. Thus, the “find” behavior of the function cannot be observed from the graph unless you change the interval of numbers for the x-axis.

7. See students’ work.

8. Yes; no, they only provide plausible visual evidence.

6-8 Trigonometric Inverses and Their Graphs

Page 410 Check for Understanding

1. $y = \sin^{-1} x$ is the inverse relation of $y = \sin x$, $y = (\sin x)^{-1}$ is the function $y = \frac{1}{\sin x}$, and $y = \sin(x^{-1})$ is the function $y = \sin \frac{1}{x}$.

2. For every y value there are more than one x value. The graph of $y = \cos^{-1} x$ fails the vertical line test.

3. The domain of $y = \text{Sin } x$ is the set of real numbers between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$, inclusive, while the domain of $y = \sin x$ is the set of all real numbers. The range of both functions is the set of all real numbers between -1 and 1, inclusive.

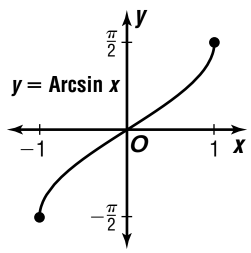
4. Restricted domains are denoted with a capital letter.

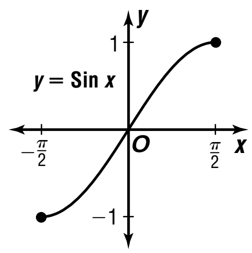
5. Akikta; there are 2 range values for each domain value between 0 and 2π. The principal values are between 0 and π, inclusive.

6. $y = \text{Arcsin } x$

$x = \text{Arcsin } y$

$\text{Sin } x = y$ or $y = \text{Sin } x$



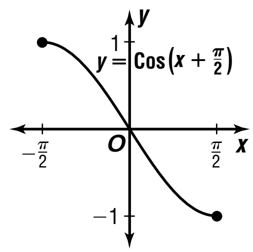


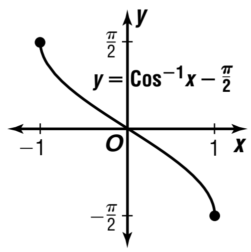
7. $y = \text{Cos}\left(x + \frac{\pi}{2}\right)$

$x = \text{Cos}\left(y + \frac{\pi}{2}\right)$

$\text{Cos}^{-1} x = y + \frac{\pi}{2}$

$y = \text{Cos}^{-1} x - \frac{\pi}{2}$





8. Let $\theta = \text{Arctan } 1$.

$\text{Tan } \theta = 1$

$\theta = \frac{\pi}{4}$

9. Let $\theta = \text{Tan}^{-1} 1$.

$\text{Tan } \theta = 1$

$\theta = \frac{\pi}{4}$

$\cos(\text{Tan}^{-1} 1) = \cos \theta$

$= \cos \frac{\pi}{4}$

$= \frac{\sqrt{2}}{2}$

10. Let $\theta = \text{Cos}^{-1}\left(\frac{\sqrt{2}}{2}\right)$.

$$\text{Cos}\,\theta = \frac{\sqrt{2}}{2}$$
$$\theta = \frac{\pi}{4}$$
$$\cos\left[\text{Cos}^{-1}\left(\frac{\sqrt{2}}{2}\right) - \frac{\pi}{2}\right] = \cos\left(\theta - \frac{\pi}{2}\right)$$
$$= \cos\left(\frac{\pi}{4} - \frac{\pi}{2}\right)$$
$$= \cos - \frac{\pi}{4}$$
$$= \frac{\sqrt{2}}{2}$$

11. true

12. false; sample answer: $x = 1$; when $x = 1$, $\text{Cos}^{-1}(-1) = \pi$, $-\text{Cos}^{-1}(1) = 0$

13a. $C = 2\pi r$
$C = 2\pi(6400)$
$C \approx 40{,}212$ km

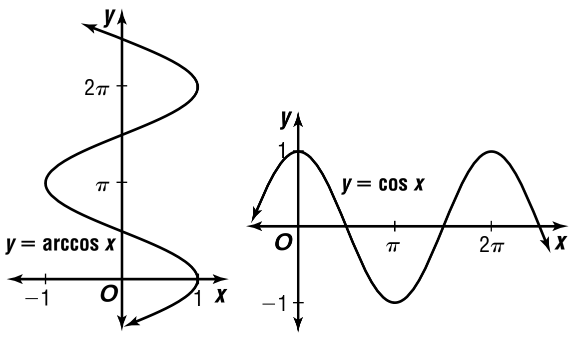
13b. $C = 40{,}212 \cos\theta$

13c.
$$C = 40{,}212 \cos\theta$$
$$3593 = 40{,}212 \cos\theta$$
$$\frac{3593}{40{,}212} = \cos\theta$$
$$\cos^{-1}\left(\frac{3593}{40{,}212}\right) = \theta$$
$1.48 \approx \theta$; about 1.48 radians

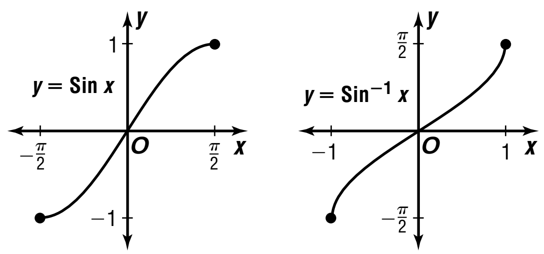
13d. $C \approx 40{,}212 \cos\theta$
$C \approx 40{,}212 \cos 0$
$C \approx 40{,}212$ km

Pages 410–412 Exercises

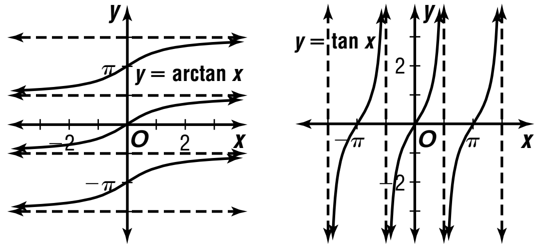
14.
$$y = \arccos x$$
$$x = \arccos y$$
$\cos x = y$ or $y = \cos x$



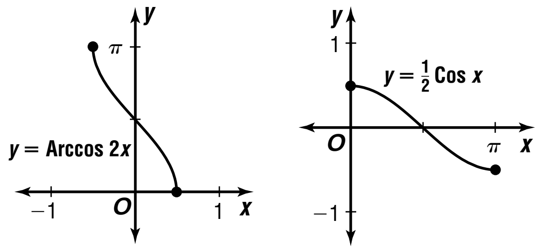
15.
$$y = \text{Sin}\,x$$
$$x = \text{Sin}\,y$$
$\text{Sin}^{-1} x = y$ or $y = \text{Sin}^{-1} x$



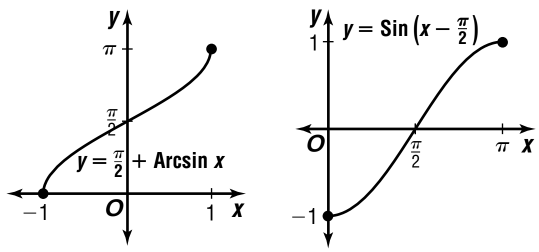
16.
$$y = \arctan x$$
$$x = \arctan y$$
$\tan x = y$ or $y = \tan x$



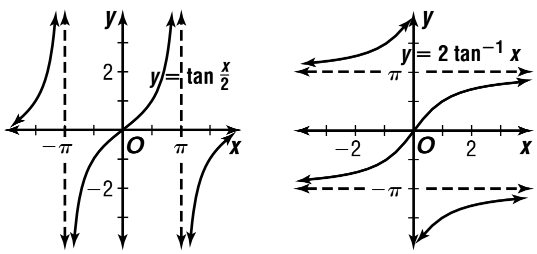
17.
$$y = \text{Arccos}\,2x$$
$$x = \text{Arccos}\,2y$$
$$\text{Cos}\,x = 2y$$
$\frac{1}{2}\text{Cos}\,x = y$ or $y = \frac{1}{2}\text{Cos}\,x$



18.
$$y = \frac{\pi}{2} + \text{Arcsin}\,x$$
$$x = \frac{\pi}{2} + \text{Arcsin}\,y$$
$$x - \frac{\pi}{2} = \text{Arcsin}\,y$$
$\text{Sin}\left(x - \frac{\pi}{2}\right) = y$ or $y = \text{Sin}\left(x - \frac{\pi}{2}\right)$



19.
$$y = \tan\frac{x}{2}$$
$$x = \tan\frac{y}{2}$$
$$\tan^{-1} x = \frac{y}{2}$$
$2\tan^{-1} x = y$; $y = 2\tan^{-1} x$



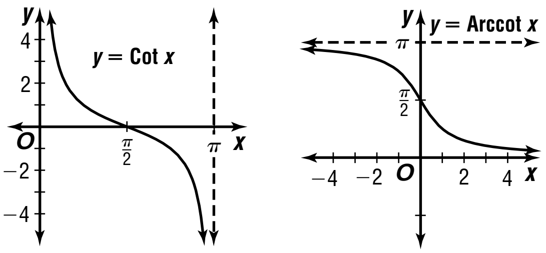
20. $y = \text{Tan}\left(x - \frac{\pi}{2}\right)$

$x = \text{Tan}\left(y - \frac{\pi}{2}\right)$

$\text{Tan}^{-1} x = y - \frac{\pi}{2}$

$\text{Tan}^{-1} x + \frac{\pi}{2} = y$

No; the inverse is $y = \text{Tan}^{-1} x + \frac{\pi}{2}$.

21. 

22. Let $\theta = \text{Sin}^{-1} 0$.
$\text{Sin}\ \theta = 0$
$\theta = 0$

23. Let $\theta = \text{Arccos}\ 0$.
$\text{Cos}\ \theta = 0$
$\theta = \frac{\pi}{2}$

24. Let $\theta = \text{Tan}^{-1} \frac{\sqrt{3}}{3}$.
$\text{Tan}\ \theta = \frac{\sqrt{3}}{3}$
$\theta = \frac{\pi}{6}$

25. If $y = \tan \frac{\pi}{4}$, then $y = 1$.

$\text{Sin}^{-1}\left(\frac{\tan \pi}{4}\right) = \text{Sin}^{-1} y$
$= \text{Sin}^{-1} 1$
$= \frac{\pi}{2}$

26. If $y = \text{Cos}^{-1} \frac{\sqrt{2}}{2}$, then $y = \frac{\pi}{4}$.

$\sin\left(2\ \text{Cos}^{-1} \frac{\sqrt{2}}{2}\right) = \sin(2y)$
$= \sin\left(2 \cdot \frac{\pi}{4}\right)$
$= \sin \frac{\pi}{2}$
$= 1$

27. If $y = \text{Tan}^{-1} \sqrt{3}$, then $y = \frac{\pi}{3}$.

$\cos(\text{Tan}^{-1} \sqrt{3}) = \cos y$
$= \cos \frac{\pi}{3}$
$= \frac{1}{2}$

28. Let $\alpha = \text{Tan}^{-1} 1$ and $\beta = \text{Sin}^{-1} 1$.

$\text{Tan}\ \alpha = 1$ $\qquad$ $\text{Sin}\ \beta = 1$

$\alpha = \frac{\pi}{4}$ $\qquad$ $\beta = \frac{\pi}{2}$

$\cos(\text{Tan}^{-1} 1 - \text{Sin}^{-1} 1) = \cos(\alpha - \beta)$
$= \cos\left(\frac{\pi}{4} - \frac{\pi}{2}\right)$
$= \cos -\frac{\pi}{4}$
$= \frac{\sqrt{2}}{2}$

29. Let $\alpha = \text{Cos}^{-1} 0$ and $\beta = \text{Sin}^{-1} \frac{1}{2}$.

$\text{Cos}\ \alpha = 0$ $\qquad$ $\text{Sin}\ \beta = \frac{1}{2}$

$\alpha = \frac{\pi}{2}$ $\qquad$ $\beta = \frac{\pi}{6}$

$\cos\left(\text{Cos}^{-1} 0 + \text{Sin}^{-1}\frac{1}{2}\right) = \cos(\alpha + \beta)$
$= \cos\left(\frac{\pi}{2} + \frac{\pi}{6}\right)$
$= \cos \frac{2\pi}{3}$
$= -\frac{1}{2}$

30. Let $\alpha = \text{Sin}^{-1} 1$ and $\beta = \text{Cos}^{-1} \frac{1}{2}$.

$\text{Sin}\ \alpha = 1$ $\qquad$ $\text{Cos}\ \beta = \frac{1}{2}$

$\alpha = \frac{\pi}{2}$ $\qquad$ $\beta = \frac{\pi}{3}$

$\sin\left(\text{Sin}^{-1} 1 - \text{Cos}^{-1} \frac{1}{2}\right) = \sin(\alpha - \beta)$
$= \sin\left(\frac{\pi}{2} - \frac{\pi}{3}\right)$
$= \sin \frac{\pi}{6}$
$= \frac{1}{2}$

31. No; there is no angle with the sine of 2.

32. false; sample answer: $x = 2\pi$; when $x = 2\pi$, $\text{Cos}^{-1}(\cos 2\pi) = \text{Cos}^{-1} 1$, or 0, not 2π.

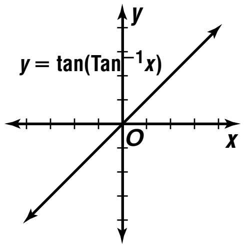
33. true

34. false; sample answer: $x = -1$; when $x = -1$, Arccos $(-1) = \pi$ and Arccos $(-(-1)) = 0$.

35. true

36. true

37. false; sample answer: $x = \frac{\pi}{2}$; when $x = \frac{\pi}{2}$, $\cos^{-1} \frac{\pi}{2}$ is undefined.

38. 

39. $y = 54.5 + 23.5 \sin\left(\frac{\pi}{6}t - \frac{2\pi}{3}\right)$

$54.5 = 54.5 + 23.5 \sin\left(\frac{\pi}{6}t - \frac{2\pi}{3}\right)$

$0 = 23.5 \sin\left(\frac{\pi}{6}t - \frac{2\pi}{3}\right)$

$0 = \sin\left(\frac{\pi}{6}t - \frac{2\pi}{3}\right)$

$\sin^{-1} 0 = \frac{\pi}{6}t - \frac{2\pi}{3}$

$0 = \frac{\pi}{6}t - \frac{2\pi}{3}$ or $\pi = \frac{\pi}{6}t - \frac{2\pi}{3}$

$\frac{2\pi}{3} = \frac{\pi}{6}t$ $\qquad$ $\frac{5\pi}{3} = \frac{\pi}{6}t$

$4 = t$ $\qquad$ $10 = t$

April and October

40. $P = VI\ \text{Cos}\ \theta$

$7.3 = 122(0.62)\ \text{Cos}\ \theta$

$0.0965097832 \approx \text{Cos}\ \theta$

$\text{Cos}^{-1} 0.0965097832 \approx \theta$

$1.47 \approx \theta$; about 1.47 radians

41. $\frac{\pi}{4} + \pi n$, where n is an integer

42. $I = I_0 \cos^2 \theta$

$1 = 8 \cos^2 \theta$

$\frac{1}{8} = \cos^2 \theta$

$\sqrt{\frac{1}{8}} = \cos \theta$

$\cos^{-1} \sqrt{\frac{1}{8}} = \theta$

$1.21 \approx \theta$; about 1.21 radians

43a. 6:18 + 12:24 = 18:42 or 6:42 P.M.

43b. 12.4 h

43c. $A = \frac{7.05 - (-0.30)}{2}$

$A = 3.675$ ft

43d. $A = \pm 3.675$ $\quad \frac{2\pi}{k} = 12.4 \quad h = \frac{7.05 + (-0.30)}{2}$

$k = \frac{\pi}{6.2} \quad h = 3.375$

6:18 = 6.3 h

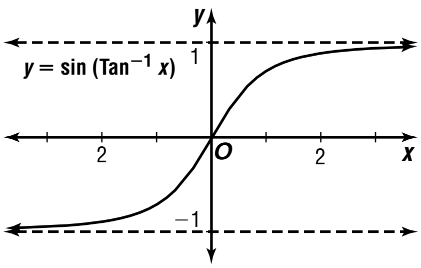
$$y = 3.675 \sin\left(\frac{\pi}{6.2}t + c\right) + 3.375$$
$$7.05 = 3.675 \sin\left(\frac{\pi}{6.2} \cdot 6.3 + c\right) + 3.375$$
$$3.675 = 3.675 \sin\left(\frac{6.3\pi}{6.2} + c\right)$$
$$1 = \sin\left(\frac{6.3\pi}{6.2} + c\right)$$
$$\sin^{-1} 1 = \frac{6.3\pi}{6.2} + c$$
$$\sin^{-1} 1 - \frac{6.3\pi}{6.2} = c$$
$$-1.621467176 \approx c$$

Sample answer:
$y = 3.375 + 3.675 \sin\left(\frac{\pi}{6.2}t - 1.62\right)$

43e.
$$y = 3.375 + 3.675 \sin\left(\frac{\pi}{6.2}t - 1.62\right)$$
$$6 = 3.375 + 3.675 \sin\left(\frac{\pi}{6.2}t - 1.62\right)$$
$$2.625 = 3.675 \sin\left(\frac{\pi}{6.2}t - 1.62\right)$$
$$\frac{2.625}{3.675} = \sin\left(\frac{\pi}{6.2}t - 1.62\right)$$
$$\sin^{-1}\left(\frac{2.625}{3.675}\right) = \frac{\pi}{6.2}t - 1.62$$
$$\sin^{-1}\left(\frac{2.625}{3.675}\right) + 1.62 = \frac{\pi}{6.2}t$$
$$\frac{6.2}{\pi}\left(\sin^{-1}\left(\frac{2.625}{3.675}\right) + 1.62\right) = t$$
$$4.767243867 \approx t$$

$0.767243867 \times 60 \approx 46.03463204$;
Sample answer: about 4:46 A.M.

44.



45a. $\theta = \cos^{-1}\frac{D - d}{2c}$

$\theta = \cos^{-1}\frac{6 - 4}{2(10)}$

$\theta \approx 1.47$ radians

45b. $L = \pi D + (d - D)\theta + 2C \sin\theta$
$L \approx \pi(6) + (4 - 6)1.47 + 2(10) \sin 1.47$
$L \approx 35.81$ in.

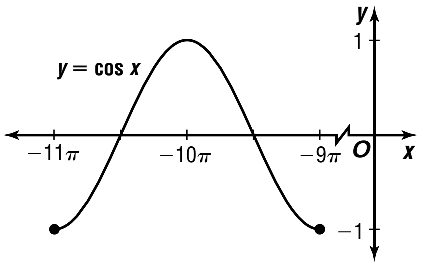
46. πn, where n is an integer

47. $|A| = 5 \quad \frac{2\pi}{k} = 3\pi \quad -\frac{c}{\frac{2}{3}} = -\pi \quad h = -8$

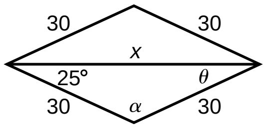
$A = \pm 5 \quad k = \frac{2}{3} \quad c = \frac{2\pi}{3}$

$y = \pm 5 \sin\left(\frac{2}{3}\theta + \frac{2\pi}{3}\right) - 8$

48.



49.

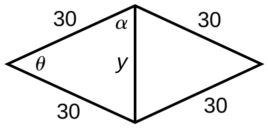


$\theta = 25°$
$\alpha = 180° - (25° + 25°)$ or $130°$

$\frac{30}{\sin 25°} = \frac{x}{\sin 130°}$

$x = \frac{30 \sin 130°}{\sin 25°}$

$x \approx 54.4$ units



$\theta = 2(25°)$ or $50°$
$\alpha = \frac{1}{2}(180° - 50°)$ or $65°$

$\frac{30}{\sin 65°} = \frac{y}{\sin 50°}$

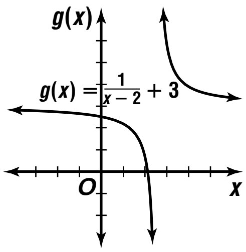
$y = \frac{30 \sin 50°}{\sin 65°}$

$y \approx 25.4$ units

50. $210° - 180° = 30°$

51. p: $\pm 1, \pm 2, \pm 3, \pm 6$
q: $\pm 1, \pm 2$
$\frac{p}{q}$: $\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}$

52.



decreasing for $x < 2$ and $x > 2$

53. $[f \circ g](x) = f(g(x))$
$= f(3x)$
$= (3x)^3 - 1$
$= 27x^3 - 1$

$[g \circ f](x) = g(f(x))$
$= g(x^3 - 1)$
$= 3(x^3 - 1)$
$= 3x^3 - 3$

54. $D = 4, F = 6, G = 7, H = 8$
value: $(4 + 6 + 7 + 8)4 = (25)4$ or 100
The correct choice is D.

Chapter 6 Study Guide and Assessment

Page 413 Understanding and Using the Vocabulary

1. radian **2.** angular
3. the same **4.** amplitude
5. angle **6.** phase
7. radian **8.** frequency
9. sunusoidal **10.** domain

Pages 414–416 Skills and Concepts

11. $60° = 60° \times \frac{\pi}{180°}$
$= \frac{\pi}{3}$

12. $-75° = -75° \times \frac{\pi}{180°}$
$= -\frac{5\pi}{12}$

13. $240° = 240° \times \frac{\pi}{180°}$
$= \frac{4\pi}{3}$

14. $\frac{5\pi}{6} = \frac{5\pi}{6} \times \frac{180°}{\pi}$
$= 150°$

15. $-\frac{7\pi}{4} = -\frac{7\pi}{4} \times \frac{180°}{\pi}$
$= -315°$

16. $2.4 = 2.4 \times \frac{180°}{\pi}$
$= 137.5°$

17. $s = r\theta$
$s = 15\left(\frac{3\pi}{4}\right)$
$s \approx 35.3$ cm

18. $75° = 75° \times \frac{\pi}{180°}$
$= \frac{5\pi}{12}$
$s = r\theta$
$s = 15\left(\frac{5\pi}{12}\right)$
$s \approx 19.6$ cm

19. $150° = 150° \times \frac{\pi}{180°}$
$= \frac{5\pi}{6}$
$s = r\theta$
$s = 15\left(\frac{5\pi}{6}\right)$
$s \approx 39.3$ cm

20. $s = r\theta$
$s = 15\left(\frac{\pi}{5}\right)$
$s \approx 9.4$ cm

21. $5 \times 2\pi = 10\pi$ or about 31.4 radians
22. $3.8 \times 2\pi = 7.6\pi$ or about 23.9 radians
23. $50.4 \times 2\pi = 100.8\pi$ or about 316.7 radians
24. $350 \times 2\pi = 700\pi$ or about 2199.1 radians

25. $1.8 \times 2\pi = 3.6\pi$
$\omega = \frac{\theta}{t}$
$\omega = \frac{3.6\pi}{5}$
$\omega \approx 2.3$ radians/s

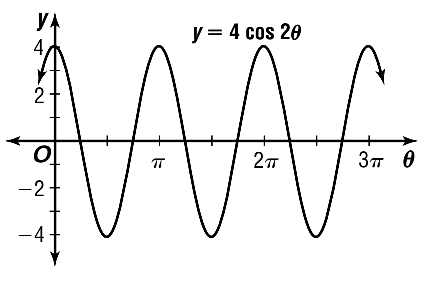
26. $3.6 \times 2\pi = 7.2\pi$
$\omega = \frac{\theta}{t}$
$\omega = \frac{7.2\pi}{2}$
$\omega \approx 11.3$ radians/min

27. $15.4 \times 2\pi = 30.8\pi$
$\omega = \frac{\theta}{t}$
$\omega = \frac{30.8\pi}{15}$
$\omega \approx 6.5$ radians/s

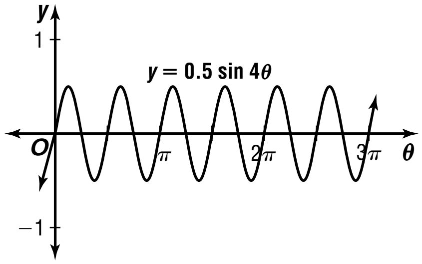
28. $50 \times 2\pi = 100\pi$
$\omega = \frac{\theta}{t}$
$\omega = \frac{100\pi}{12}$
$\omega \approx 26.2$ radians/min

29. -1 **30.** 0 **31.** 1 **32.** 0

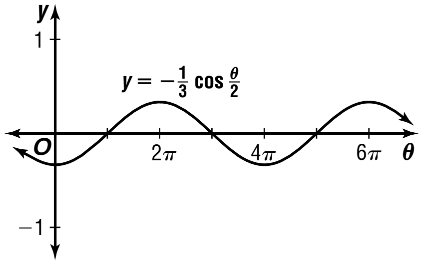
33. $|4| = 4; \frac{2\pi}{2} = \pi$



34. $|0.5| = 0.5; \frac{2\pi}{4} = \frac{\pi}{2}$



35. $\left|-\frac{1}{3}\right| = \frac{1}{3}; \frac{2\pi}{\frac{1}{2}} = 4\pi$



36. $|A| = 4$ $\quad \frac{2\pi}{k} = \frac{\pi}{2}$ $\quad -\frac{c}{4} = -2\pi$ $\quad h = -1$
$A = \pm 4$ $\quad k = 4$ $\quad c = 8\pi$
$y = \pm 4 \sin (4\theta + 8\pi) - 1$

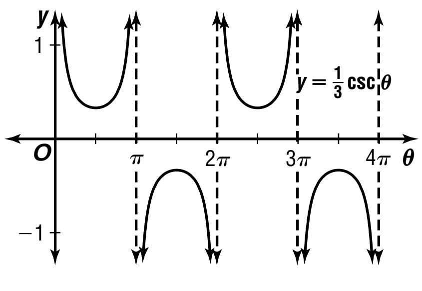
37. $|A| = 0.5$ $\quad \frac{2\pi}{k} = \pi$ $\quad -\frac{c}{2} = \frac{\pi}{3}$ $\quad h = 3$
$A = \pm 0.5$ $\quad k = 2$ $\quad c = -\frac{2\pi}{3}$
$y = \pm 0.5 \sin \left(2\theta - \frac{2\pi}{3}\right) + 3$

38. $|A| = \frac{3}{4}$ $\quad \frac{2\pi}{k} = \frac{\pi}{4}$ $\quad -\frac{c}{8} = 0$ $\quad h = 5$
$A = \pm\frac{3}{4}$ $\quad k = 8$ $\quad c = 0$
$y = \pm\frac{3}{4} \cos 8\theta + 5$

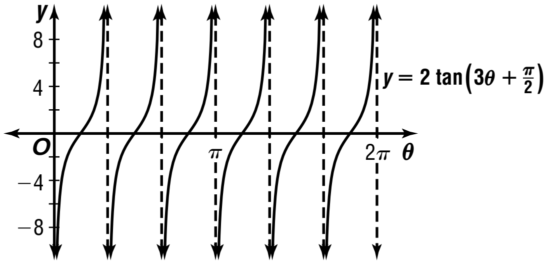
39. $A = \frac{120 - 80}{2}$ $\quad \frac{2\pi}{k} = 1$ $\quad h = \frac{120 + 80}{2}$
$A = 20$ $\quad k = 2\pi$ $\quad h = 100$
$y = 20 \sin 2\pi t + 100$

40. $A = \frac{130 - 100}{2}$ $\quad \frac{2\pi}{k} = 1$ $\quad h = \frac{130 + 100}{2}$
$A = 15$ $\quad k = 2\pi$ $\quad h = 115$
$y = 15 \sin 2\pi t + 115$

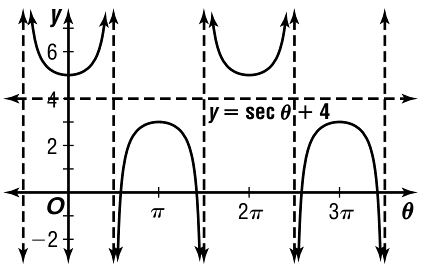
41. period: $\frac{2\pi}{1}$ or 2π, no phase shift, no vertical shift



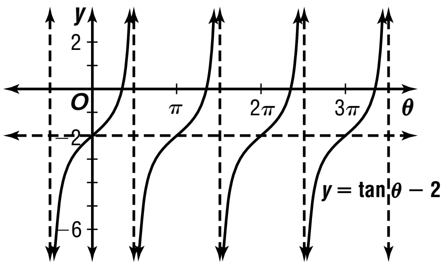
42. $\frac{\pi}{3}$; $-\frac{\frac{\pi}{2}}{3} = -\frac{\pi}{6}$; no vertical shift



43. vertical shift: 4



44. vertical shift: −2



45. Let $\theta = \text{Arctan } -1$.
$\text{Tan } \theta = -1$
$\theta = -\frac{\pi}{4}$

46. Let $\theta = \text{Sin}^{-1} 1$.
$\text{Sin } \theta = 1$
$\theta = \frac{\pi}{2}$

47. If $y = \tan \frac{\pi}{4}$, then $y = 1$.
$\text{Cos}^{-1}\left(\tan \frac{\pi}{4}\right) = \text{Cos}^{-1} y$
$= \text{Cos}^{-1} 1$
Let $\theta = \text{Cos}^{-1} 1$.
$\text{Cos } \theta = 1$
$\theta = 0$

48. If $y = \text{Sin}^{-1} \frac{\sqrt{3}}{2}$, then $y = \frac{\pi}{3}$.
$\sin\left(\text{Sin}^{-1} \frac{\sqrt{3}}{2}\right) = \sin y$
$= \sin \frac{\pi}{3}$
$= \frac{\sqrt{3}}{2}$

49. Let $\alpha = \text{Arctan } \sqrt{3}$ and $\beta = \text{Arcsin } \frac{1}{2}$.
$\text{Tan } \alpha = \sqrt{3}$ $\quad$ $\text{Sin } \beta = \frac{1}{2}$
$\alpha = \frac{\pi}{3}$ $\quad$ $\beta = \frac{\pi}{6}$
$\cos\,(\text{Arctan } \sqrt{3} + \text{Arcsin } \frac{1}{2} = \cos\,(\alpha + \beta)$
$= \cos\left(\frac{\pi}{3} + \frac{\pi}{6}\right)$
$= \cos \frac{\pi}{2}$
$= 0$

Page 417 Applications and Problem Solving

50a. $A = 11.5$ $\quad$ $2\frac{\pi}{k} = 12$ $\quad$ $-\frac{c}{\frac{\pi}{6}} = 3$ $\quad$ $h = 64$
$k = \frac{\pi}{6}$ $\quad$ $c = -\frac{\pi}{2}$
$y = 11.5 \sin\left(\frac{\pi}{6}t - \frac{\pi}{2}\right) + 64$

50b. April: month 4
$y = 11.5 \sin\left(\frac{\pi}{6}t - \frac{\pi}{2}\right) + 64$
$y = 11.5 \sin\left(\frac{\pi}{6} \cdot 4 - \frac{\pi}{2}\right) + 64$
$y = 69.75$; about 69.8°

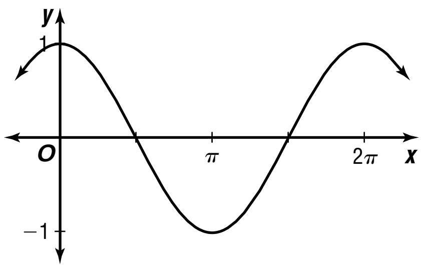
50c. July: month 7
$y = 11.5 \sin\left(\frac{\pi}{6}t - \frac{\pi}{2}\right) + 64$
$y = 11.5 \sin\left(\frac{\pi}{6} \cdot 7 - \frac{\pi}{2}\right) + 64$
$y \approx 74.0°$

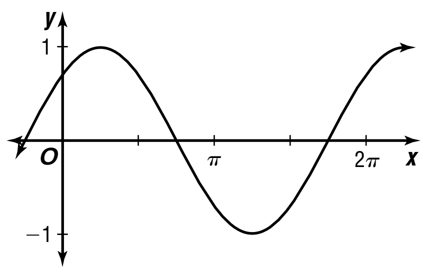
51.
$B = \frac{F}{IL \sin \theta}$
$0.04 = \frac{0.2}{5.0(1) \sin \theta}$
$0.04(5.0(1) \sin \theta) = 0.2$
$\sin \theta = \frac{0.2}{0.04(5.0)(1)}$
$\sin \theta = 1$
$\theta = \frac{\pi}{2}$

Page 417 Open-Ended Assessment

1. $A = \frac{1}{2}r^2\theta$
$26.2 = \frac{1}{2}r^2\theta$
Sample answer: $r = 5$ in., $\theta = \frac{2\pi}{3}$

2a. Sample answer: If the graph does not cross the y-axis at 1, the graph has been translated. The first graph has not been translated and the second graph has been translated.



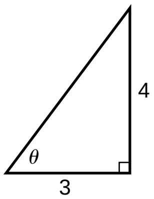


2b. Sample answer: If the equation does not have the form $y = A \cos k\theta$, the graph has been translated. The graph of $y = 2 \cos 2\theta$ has not been translated. The graph of $y = 2 \cos (2\theta + \pi) - 3$ has been translated vertically and horizontally.

Chapter 6 SAT & ACT Preparation

Page 419 SAT and ACT Practice

1. Since there is no diagram, draw one. Sketch a right triangle and mark the information given.



Notice that this is one of the "special" right triangles. Its sides are 3-4-5. So the hypotenuse is 5. The sine is opposite over hypotenuse (SOH).

$\sin \theta = \frac{4}{5}$

The correct choice is B.

2. Let x be the smaller integer. The numbers are two consecutive odd integers. So, the larger integer is 2 more than the first integer. Represent the larger integer by $x + 2$. Write an equation that says that the sum of these two integers is 56. Then solve for x.

$$x + (x + 2) = 56$$
$$2x + 2 = 56$$
$$2x = 54$$
$$x = 27$$

Be sure to read the question carefully. It asks for the value of the larger integer. The smaller integer is 27 and the larger integer is 29.

The correct choice is C.

3. Factor the numerator.

$a^2 - b^2 = (a + b)(a - b)$

$$\frac{(\sin \theta + \cos \theta)(\sin \theta - \cos \theta)}{\sin \theta - \cos \theta} = \sin \theta + \cos \theta$$

The correct choice is B.

4. First find the coordinates of point B. Notice that there are two right triangles. One has a hypotenuse of length 15 and a side of length 12. This is a 3-4-5 right triangle. The coordinates of point B are (9, 12).

Since point A has coordinates (0, 0), each point on side AB must have coordinates in the ratio of 9 to 12 or 3 to 4.

The only point among the answer choices that has this ratio of coordinates is (6, 8).

A slightly different way of solving this problem is to write the equation of the line containing points A and B.

$y = \frac{12}{9}x$

Then test each point to see whether it makes the equation a true statement.

You could also plot each point on the figure and see which point seems to lie on the line segment.

The correct choice is E.

5. Factor the polynomial on the left side of the equation.

$$x^2 - 2x - 8 = 0$$
$$(x - 4)(x + 2) = 0$$

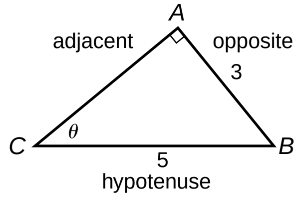
If either of the two factors equals 0, then the statement is true. Set each factor equal to 0 and solve for x.

$x - 4 = 0$ or $x + 2 = 0$

$x = 4$ $\quad$ $x = -2$

The solutions of the equation are 4 and −2. To find the sum of the solutions, add 4 + −2 = 2. The correct choice is D.

6. You may want to label the triangle with *opposite, adjacent,* and *hypotenuse.*



To find cos θ, you need to know the length of the adjacent side. Notice that the hypotenuse is 5 and one side is 3, so this is a 3-4-5 right triangle. The adjacent side is 4 units.

Use the ratio for cos θ.

$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{5}$

The correct choice is C.

7. Look at the powers of the variables in the equation. There is an x^2 term, an x term, and a y term, but *no* y^2 term. It cannot represent a line, because of the x^2 term. It cannot represent a circle or an ellipse or a hyperbola because there is no y^2 term. So, it must represent a parabola.

The general form of the equation of a parabola is $y = a(x - h)^2 + k$. The correct choice is A.

8. Factor each of the numerators and determine if the resulting expression could be an integer, that is, the numerator is a multiple of the denominator.

I $\quad \frac{16n + 16}{n + 1} = \frac{16(n + 1)}{n + 1} = 16$; an integer

II $\quad \frac{16n + 16}{16n} = \frac{16(n + 1)}{16n} = \frac{n + 1}{n}$; not an integer

III $\quad \frac{16n^2 + n}{16n} = \frac{n(16n + 1)}{16n} = \frac{16n + 1}{16}$; not an integer

Only expression I is an integer.

The correct choice is A.

9. Since $x > 1$, $1 - x < 0$. So $x^{1 - x} = \frac{1}{x^{x - 1}}$.

Since $x > 1$, $x^{x - 1} > 1$. So $\frac{1}{x^{x - 1}} < 1$.

The correct choice is D.

10. Notice that the triangles are not necessarily isosceles. In $\triangle ADC$, the sum of the angles is 180°, so $m\angle CAD + m\angle ACD = 80$. Since segment AD bisects $\angle BAC$, $m\angle BAD + m\angle CAD$. Similarly, $m\angle BAC = m\angle ACD$. So, $m\angle BAD + m\angle BCD = 80$.

Add the two equations. $m\angle CAD + m\angle BAD + m\angle ACD + m\angle BCD = 160$, so two of the angles in $\triangle ABC$ have the combined measure of 160°. Therefore, the third angle in this triangle, $\angle B$, must measure 20°. The correct answer is 20.